

Trig Substitutions

$$\sqrt{a^2 + x^2}$$

use $x = a \tan \theta$

$$\sqrt{a^2 - x^2}$$

use $x = a \sin \theta$ or $x = a \cos \theta$

$$\sqrt{x^2 - a^2}$$

use $x = a \sec \theta$

$$\text{e.g. (i)} \int \frac{1}{\sqrt{a^2 + x^2}} dx$$

$$= \int \frac{a \sec^2 \theta d\theta}{\sqrt{a^2 + a^2 \tan^2 \theta}}$$

$$= \int \frac{a \sec^2 \theta d\theta}{\sqrt{a^2 \sec^2 \theta}}$$

$$= \int \sec \theta d\theta$$

$$= \log(\sec \theta + \tan \theta) + c$$

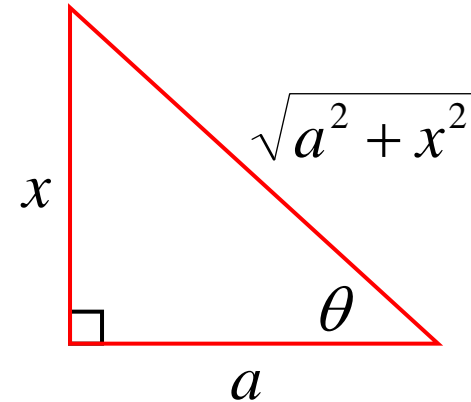
$$= \log\left(\frac{\sqrt{a^2 + x^2}}{a} + \frac{x}{a}\right) + c$$

$$= \log(\sqrt{a^2 + x^2} + x) - \log a + c$$

$$= \log(\sqrt{a^2 + x^2} + x) + c$$

$$x = a \tan \theta$$

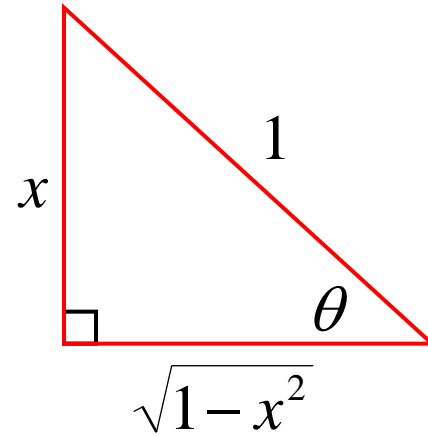
$$dx = a \sec^2 \theta d\theta$$



$$\begin{aligned} (ii) \int \frac{x}{\sqrt{1-x^2}} dx &= \int \frac{\sin \theta \cos \theta d\theta}{\sqrt{1-\sin^2 \theta}} \\ &= \int \frac{\sin \theta \cos \theta d\theta}{\sqrt{\cos^2 \theta}} \\ &= \int \sin \theta d\theta \\ &= -\cos \theta + c \\ &= -\sqrt{1-x^2} + c \end{aligned}$$

$$x = \sin \theta$$

$$dx = \cos \theta d\theta$$



$$(iii) \int \sqrt{x^2 + 3} dx$$

$$= \int (\sqrt{3} \sec \theta) (\sqrt{3} \sec^2 \theta) d\theta$$

$$= 3 \int \sec^3 \theta d\theta$$

$$= 3 \int \sec \theta \sec^2 \theta d\theta$$

$$= 3 \sec \theta \tan \theta - 3 \int \sec \theta \tan^2 \theta d\theta$$

$$= 3 \sec \theta \tan \theta - 3 \int \sec^3 \theta d\theta + 3 \int \sec \theta$$

$$= 3 \sec \theta \tan \theta - 3 \int \sec^3 \theta d\theta + 3 \log(\sec \theta + \tan \theta)$$

$$= 3 \frac{\sqrt{x^2 + 3}}{\sqrt{3}} \cdot \frac{x}{\sqrt{3}} - \int \sqrt{x^2 + 3} dx + 3 \log \left(\frac{\sqrt{x^2 + 3}}{\sqrt{3}} + \frac{x}{\sqrt{3}} \right)$$

$$x = \sqrt{3} \tan \theta$$

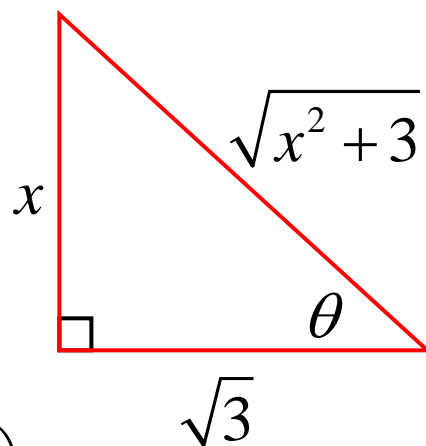
$$dx = \sqrt{3} \sec^2 \theta d\theta$$

$$u = \sec \theta$$

$$v = \tan \theta$$

$$du = \sec \theta \tan \theta d\theta$$

$$dv = \sec^2 \theta d\theta$$



$$\therefore 2 \int \sqrt{x^2 + 3} dx = x\sqrt{x^2 + 3} + 3 \log \left(\frac{\sqrt{x^2 + 3}}{\sqrt{3}} + \frac{x}{\sqrt{3}} \right) + c$$

$$\int \sqrt{x^2 + 3} dx = \frac{x\sqrt{x^2 + 3}}{2} + \frac{3}{2} \log \left(\frac{\sqrt{x^2 + 3}}{\sqrt{3}} + \frac{x}{\sqrt{3}} \right) + c$$

$$\int \sqrt{x^2 + 3} dx = \frac{x\sqrt{x^2 + 3}}{2} + \frac{3}{2} \log \left(\sqrt{x^2 + 3} + x \right) + c$$

Exercise 2E; 1, 2, 3, 5, 6, 7, 9, 13, 17, 19, 20