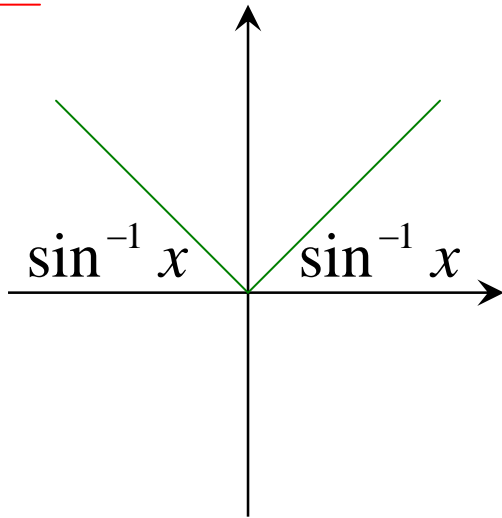


# General Solutions of Trig Equations

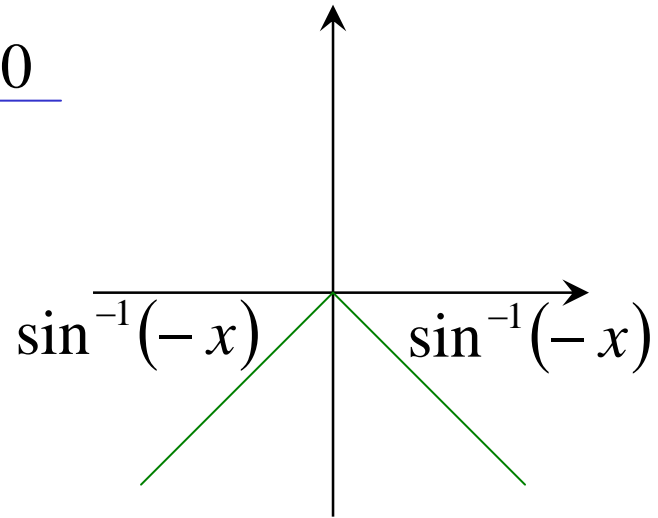
$$\sin \theta = x$$

$$x > 0$$



$$\theta = \sin^{-1} x \text{ or } \pi - \sin^{-1} x$$

$$x < 0$$



$$\theta = \pi + \sin^{-1}(-x) \text{ or } 2\pi - \sin^{-1}(-x)$$

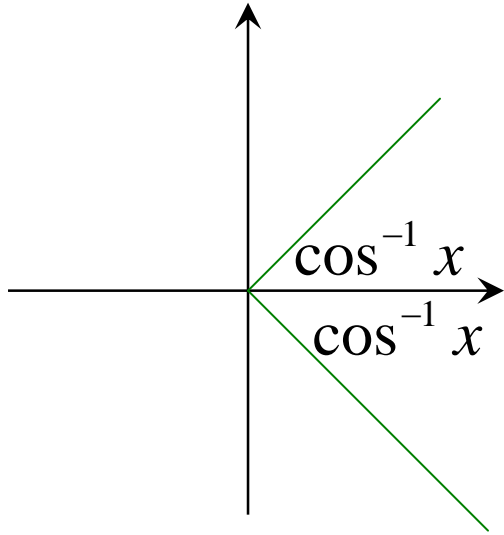
$$\theta = \pi - \sin^{-1} x \text{ or } 2\pi + \sin^{-1} x$$

$$\sin \theta = x$$

$$\theta = \pi k + (-1)^k \sin^{-1} x \quad \text{where } k \text{ is an integer}$$

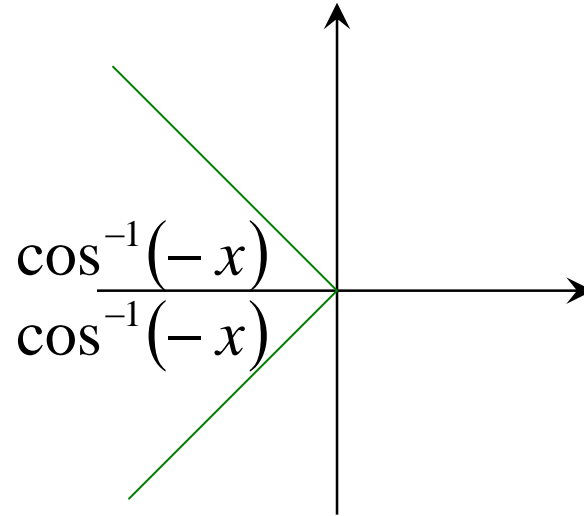
$$\underline{\cos \theta = x}$$

$$\underline{x > 0}$$



$$\theta = \cos^{-1} x \text{ or } 2\pi - \cos^{-1} x$$

$$\underline{x < 0}$$



$$\theta = \pi - \cos^{-1}(-x) \text{ or } \pi + \cos^{-1}(-x)$$

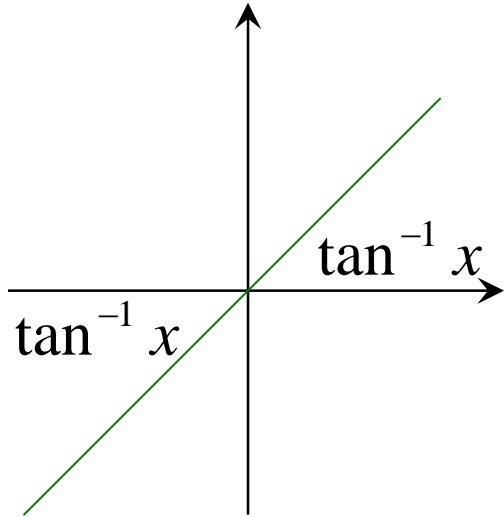
$$\theta = \pi - (\pi - \cos^{-1} x) \text{ or } \pi + (\pi - \cos^{-1} x)$$
$$= \cos^{-1} x \quad \text{or } 2\pi - \cos^{-1} x$$

$$\cos \theta = x$$

$$\theta = 2\pi k \pm \cos^{-1} x \quad \text{where } k \text{ is an integer}$$

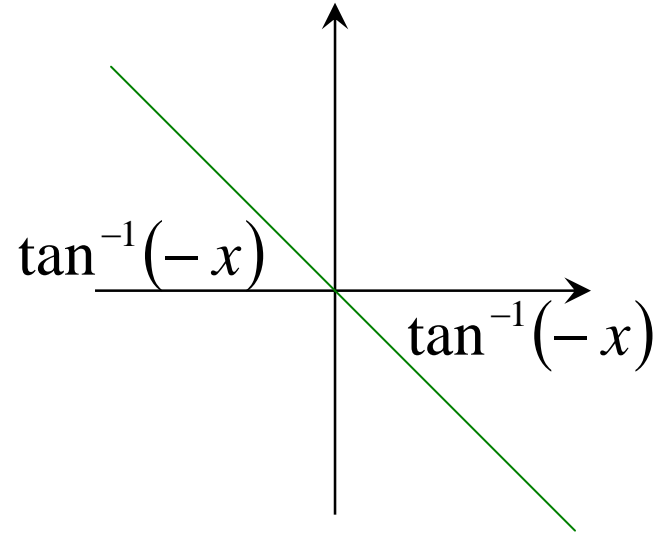
$$\underline{\tan \theta = x}$$

$$\underline{x > 0}$$



$$\theta = \tan^{-1} x \text{ or } \pi + \tan^{-1} x$$

$$\underline{x < 0}$$



$$\theta = \pi - \tan^{-1}(-x) \text{ or } 2\pi - \tan^{-1}(-x)$$

$$\theta = \pi + \tan^{-1} x \text{ or } 2\pi + \tan^{-1} x$$

$$\tan \theta = x$$

$$\theta = \pi k + \tan^{-1} x$$

where  $k$  is an integer

e.g. (i)  $\sin \theta = \frac{\sqrt{3}}{2}$

$$\theta = \pi k + (-1)^k \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

where  $k$  is an integer

$$\theta = \pi k + (-1)^k \frac{\pi}{3}$$

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If  $0 \leq \theta \leq 2\pi$

$$\theta = \frac{\pi}{3}, \pi - \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

---

(ii)  $\cos \theta = -\frac{1}{\sqrt{2}}$

$$\theta = 2\pi k \pm \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

where  $k$  is an integer

$$\theta = 2\pi k \pm \frac{3\pi}{4}$$

---

If  $0 \leq \theta \leq 2\pi$

$$\theta = \frac{3\pi}{4}, 2\pi - \frac{3\pi}{4}$$

$$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$$

---

$$(iii) \tan \theta = \frac{1}{\sqrt{3}}$$

$$\theta = \pi k + \tan^{-1} \left( \frac{1}{\sqrt{3}} \right)$$

where  $k$  is an integer

$$\theta = \pi k + \frac{\pi}{6}$$

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If  $0 \leq \theta \leq 2\pi$

$$\theta = \frac{\pi}{6}, \pi + \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{7\pi}{6}$$

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$$(iv) \sin \theta = \sin \frac{5\pi}{7}$$

$$\theta = \pi k + (-1)^k \sin^{-1} \sin \frac{5\pi}{7}$$

$$\theta = \pi k + (-1)^k \frac{2\pi}{7}$$

---

where  $k$  is an integer

$$(v) \cos 2x = \cos \frac{\pi}{9}$$

$$2x = 2\pi k \pm \cos^{-1} \cos \frac{\pi}{9}$$

$$2x = 2\pi k \pm \frac{\pi}{9}$$

$$x = \pi k \pm \frac{\pi}{18}$$

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where  $k$  is an integer

**Exercise 1F; 4 to 8 ace etc  
9 to 11  
12ac**