

# *Dynamics*

## Newton's Laws Of Motion

### *Law 1*

*“ Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter, nisi quatenus a viribus impressis cogitur statum illum mutare.”*

Everybody continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

### *Law 2*

*“ Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum rectam qua vis illa imprimitur.”*

The change of motion is proportional to the motive force impressed, and it is made in the direction of the right line in which that force is impressed.

Motive force  $\propto$  change in motion

$F \propto$  acceleration

$F = \text{constant} \times \text{acceleration}$

$\therefore F = ma$  (constant = mass)

### ***Law 3***

*“Actioni contrariam semper et aequalem esse reactionem: sive corporum duorum actions in se mutuo semper esse aequales et in partes contrarias dirigi.”*

To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed in contrary parts.

# *Acceleration*

$$\ddot{x} = \frac{d^2 x}{dt^2} \quad (\text{acceleration is a function of } t)$$

$$= \frac{dv}{dt}$$

$$= \frac{d}{dx} \left( \frac{1}{2} v^2 \right) \quad (\text{acceleration is a function of } x)$$

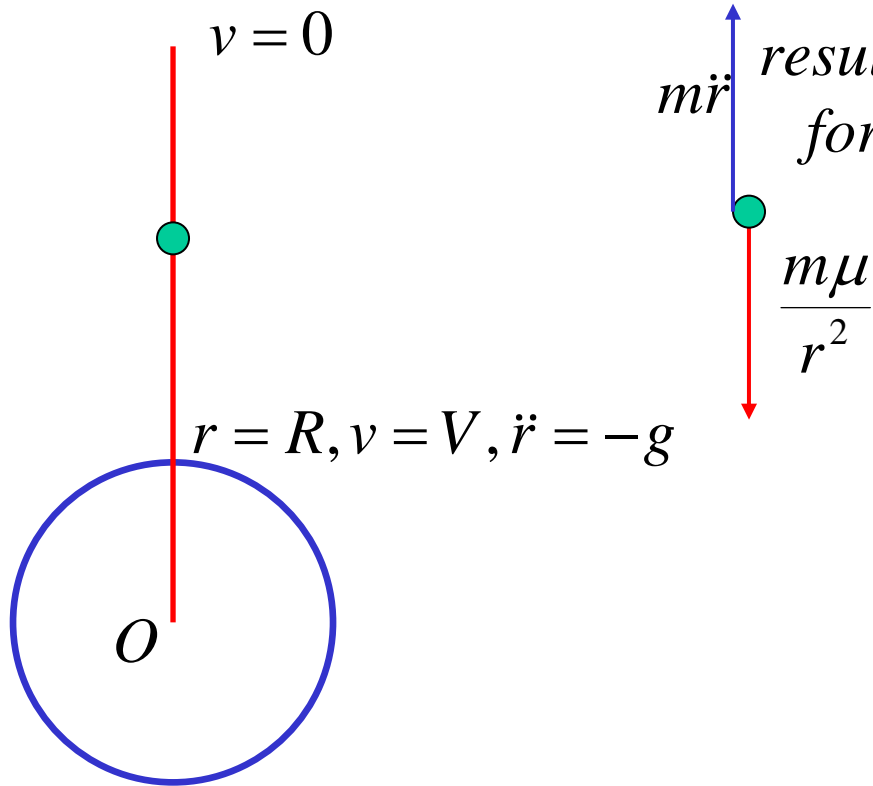
$$= v \frac{dv}{dx} \quad (\text{acceleration is a function of } v)$$

e.g. (1981)

Assume that the earth is a sphere of radius  $R$  and that, at a point  $r$  ( $\geq R$ ) from the centre of the earth, the acceleration due to gravity is proportional to  $\frac{1}{r^2}$  and is directed towards the earth's centre.

A body is projected vertically upwards from the surface of the earth with initial speed  $V$ .

(i) Prove that it will escape the earth if and only if  $V \geq \sqrt{2gR}$  where  $g$  is the magnitude of the acceleration due to gravity at the earth's surface.



$$m\ddot{r} = -\frac{m\mu}{r^2}$$

$$\ddot{r} = -\frac{\mu}{r^2}$$

when  $r = R, \ddot{r} = -g$

i.e.  $-g = \frac{-\mu}{R^2}$

$$\mu = gR^2$$

$$\ddot{r} = \frac{-gR^2}{r^2}$$

$$\frac{d}{dr} \left( \frac{1}{2} v^2 \right) = \frac{-gR^2}{r^2}$$

$$\frac{1}{2}v^2 = \frac{gR^2}{r} + c$$

$$v^2 = \frac{2gR^2}{r} + c$$

when  $r = R, v = V$

$$\text{i.e. } V^2 = \frac{2gR^2}{R} + c$$

$$c = V^2 - 2gR$$

$$v^2 = \frac{2gR^2}{r} + V^2 - 2gR$$

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If particle never stops then  $r \rightarrow \infty$

$$\begin{aligned} \lim_{r \rightarrow \infty} v^2 &= \lim_{r \rightarrow \infty} \left( \frac{2gR^2}{r} + V^2 - 2gR \right) \\ &= V^2 - 2gR \end{aligned}$$

Now  $v^2 \geq 0$

$$\therefore V^2 - 2gR \geq 0$$

$$V^2 \geq 2gR$$

Thus if  $V \geq \sqrt{2gR}$  the particle never stops,  
i.e. the particle escapes the earth.

(ii) If  $V = \sqrt{2gR}$ , prove that the time taken to rise to a height  $R$  above the earth's surface is;

$$\frac{1}{3}(4 - \sqrt{2})\sqrt{\frac{R}{g}}$$

$$v^2 = \frac{2gR^2}{r} + V^2 - 2gR$$

$$\text{If } V = \sqrt{2gR}, \quad v^2 = \frac{2gR^2}{r} + 2gR - 2gR$$

$$v^2 = \frac{2gR^2}{r}$$

$$v = \sqrt{\frac{2gR^2}{r}}$$

(cannot be -ve, as it does not return)

$$\frac{dr}{dt} = \sqrt{\frac{2gR^2}{r}}$$

$$\frac{dt}{dr} = \sqrt{\frac{r}{2gR^2}}$$

$$t = \frac{1}{\sqrt{2gR^2}} \int_R^{2R} \sqrt{r} dr \quad \left\{ \int_R^{2R} \text{ as travelling from } R \text{ to } 2R \right\}$$

$$= \frac{1}{\sqrt{2gR^2}} \left[ \frac{2}{3} r^{\frac{3}{2}} \right]_R^{2R}$$

$$= \frac{\sqrt{2}}{3\sqrt{gR^2}} (2R\sqrt{2R} - R\sqrt{R})$$

$$= \frac{\sqrt{2}}{3R\sqrt{g}} (2\sqrt{2} - 1)R\sqrt{R}$$

$$= \frac{\sqrt{R}}{3\sqrt{g}} (4 - \sqrt{2})$$

$$= \frac{1}{3} (4 - \sqrt{2}) \sqrt{\frac{R}{g}}$$

$\therefore$  time taken to rise to a height  $R$  above

earth's surface is  $\frac{1}{3} (4 - \sqrt{2}) \sqrt{\frac{R}{g}}$  seconds

**Exercise 8C; 3, 15, 19**