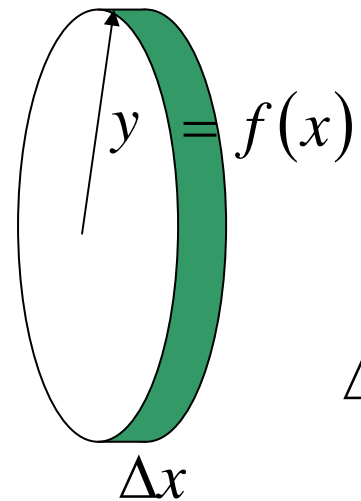
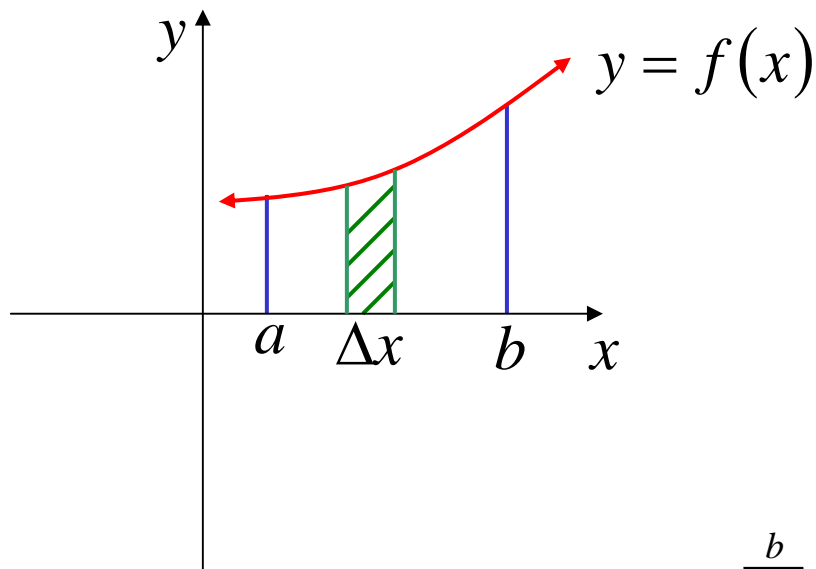


Volumes of Solids

Volumes By Discs & Washers

(slice \perp rotation)

About the x axis

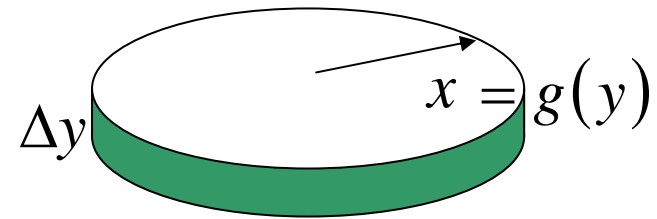
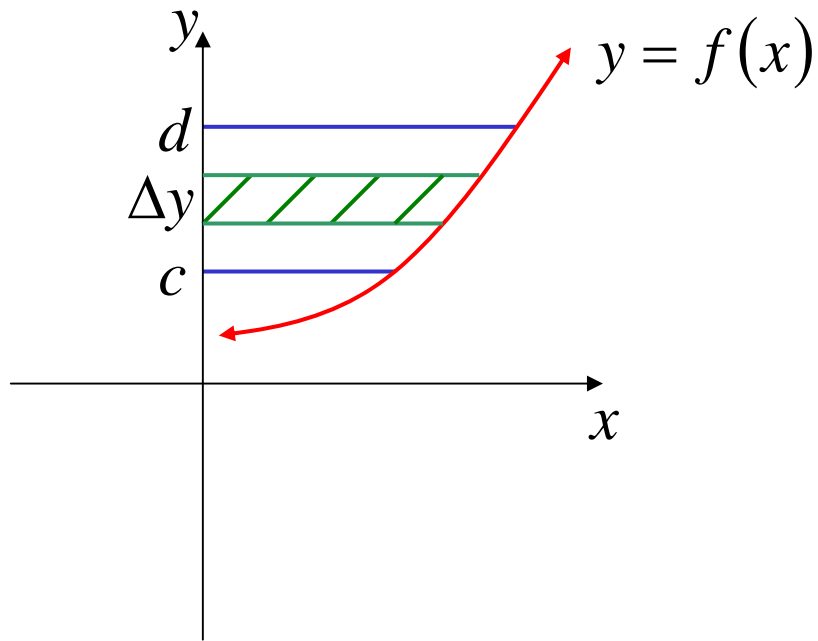


$$A(x) = \pi [f(x)]^2$$

$$\Delta V = \pi [f(x)]^2 \cdot \Delta x$$

$$\begin{aligned} V &= \lim_{\Delta x \rightarrow 0} \sum_{x=a}^b \pi [f(x)]^2 \cdot \Delta x \\ &= \pi \int_a^b [f(x)]^2 dx \end{aligned}$$

About the y axis

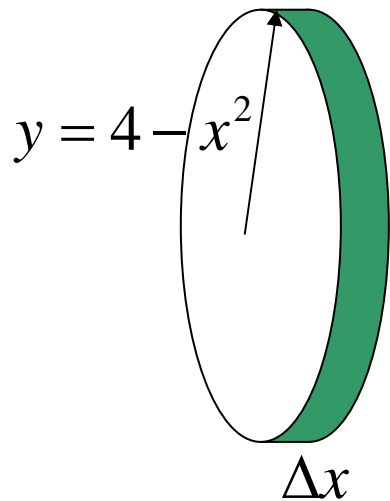
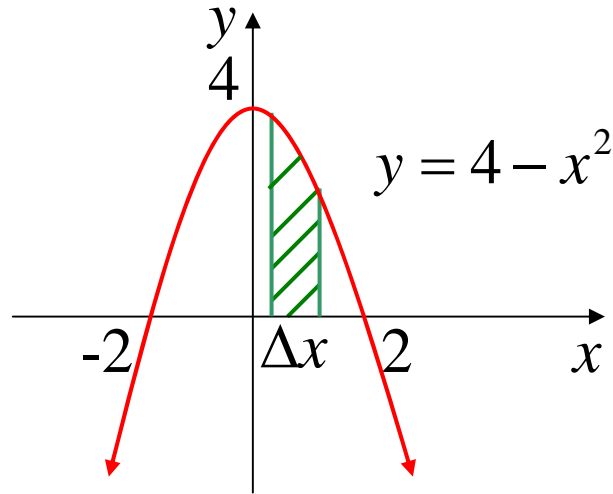


$$A(y) = \pi [g(y)]^2$$

$$\Delta V = \pi [g(y)]^2 \cdot \Delta y$$

$$\begin{aligned} V &= \lim_{\Delta y \rightarrow 0} \sum_{y=c}^d \pi [g(y)]^2 \cdot \Delta y \\ &= \pi \int_c^d [g(y)]^2 dy \end{aligned}$$

e.g. (i) Find the volume generated when the area between $y = 4 - x^2$ and the x axis is rotated about the x axis



$$A(x) = \pi(4 - x^2)^2$$

$$\Delta V = \pi(16 - 8x^2 + x^4) \cdot \Delta x$$

$$V = \lim_{\Delta x \rightarrow 0} \sum_{x=-2}^2 \pi(16 - 8x^2 + x^4) \cdot \Delta x$$

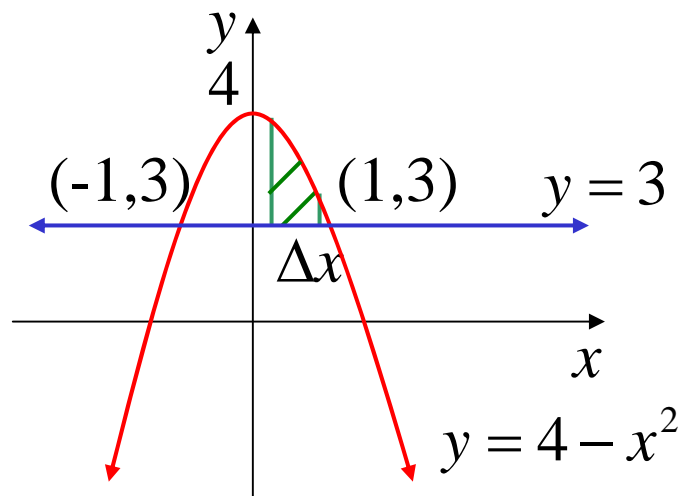
$$= 2\pi \int_0^2 (16 - 8x^2 + x^4) dx$$

$$= 2\pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5 \right]_0^2$$

$$= 2\pi \left\{ 32 - \frac{64}{3} + \frac{32}{5} - 0 \right\}$$

$$= \frac{512\pi}{15} \text{ units}^3$$

(ii) Find the volume generated when the area enclosed by $y = 4 - x^2$ and $y = 3$ is rotated about the line $y = 3$



$$y - 3 = 4 - x^2 - 3$$

$$= 1 - x^2$$

$$A(x) = \pi(1 - x^2)^2$$

$$\Delta V = \pi(1 - 2x^2 + x^4) \cdot \Delta x$$

$$V = \lim_{\Delta x \rightarrow 0} \sum_{x=-1}^1 \pi(1 - 2x^2 + x^4) \cdot \Delta x$$

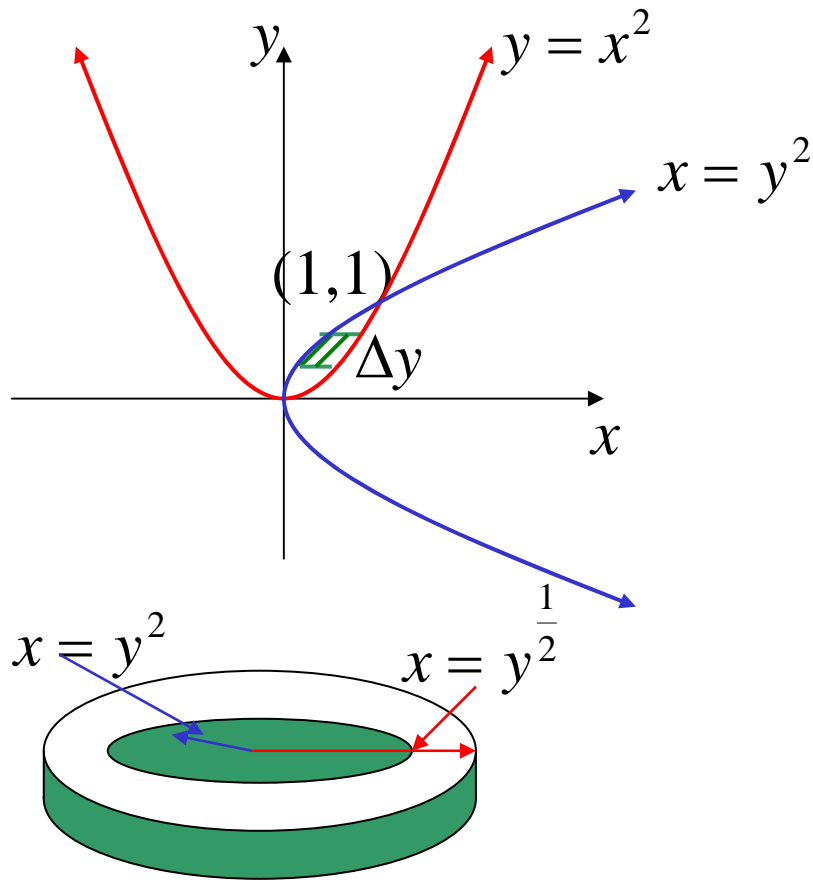
$$= 2\pi \int_0^1 (1 - 2x^2 + x^4) dx$$

$$= 2\pi \left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5 \right]_0^1$$

$$= 2\pi \left\{ 1 - \frac{2}{3} + \frac{1}{5} - 0 \right\}$$

$$= \frac{16\pi}{15} \text{ units}^3$$

(iii) The area between the curves $y = x^2$ and $x = y^2$ is rotated about the line y axis. Find the volume generated.



$$A(y) = \pi \left[\left(y^{\frac{1}{2}} \right)^2 - (y^2)^2 \right]$$

$$\Delta V = \pi (y - y^4) \cdot \Delta y$$

$$V = \lim_{\Delta y \rightarrow 0} \sum_{y=0}^1 \pi (y - y^4) \cdot \Delta y$$

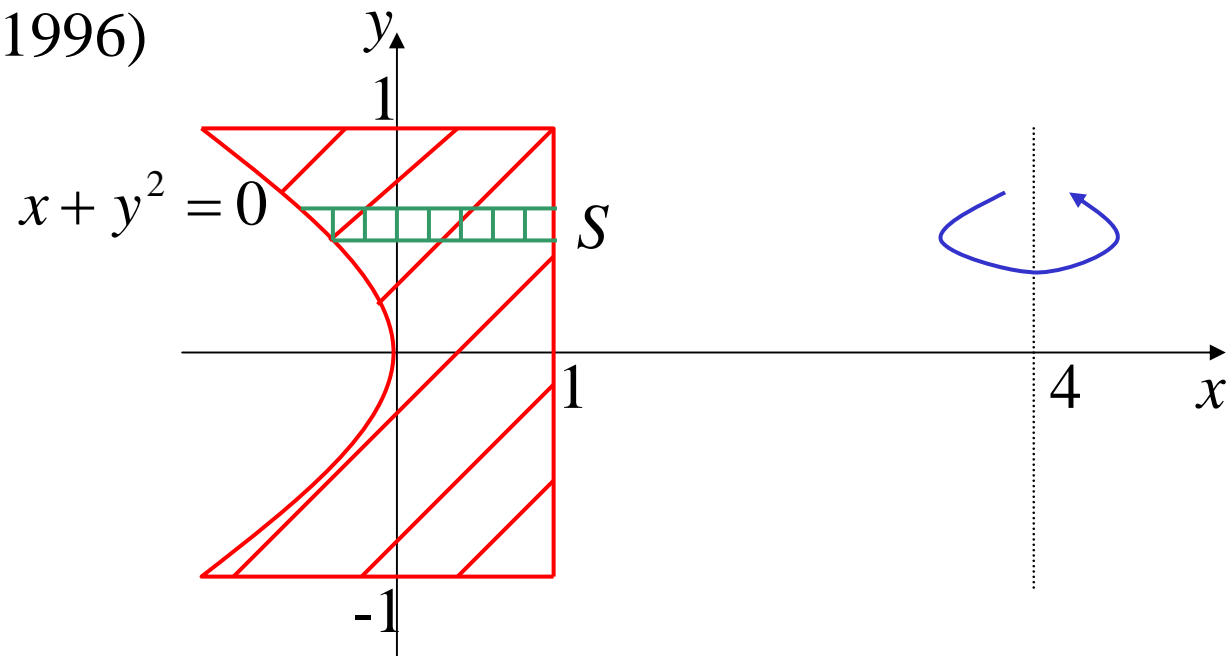
$$= \pi \int_0^1 (y - y^4) dy$$

$$= \pi \left[\frac{1}{2} y^2 - \frac{1}{5} y^5 \right]_0^1$$

$$= \pi \left\{ \frac{1}{2} - \frac{1}{5} - 0 \right\}$$

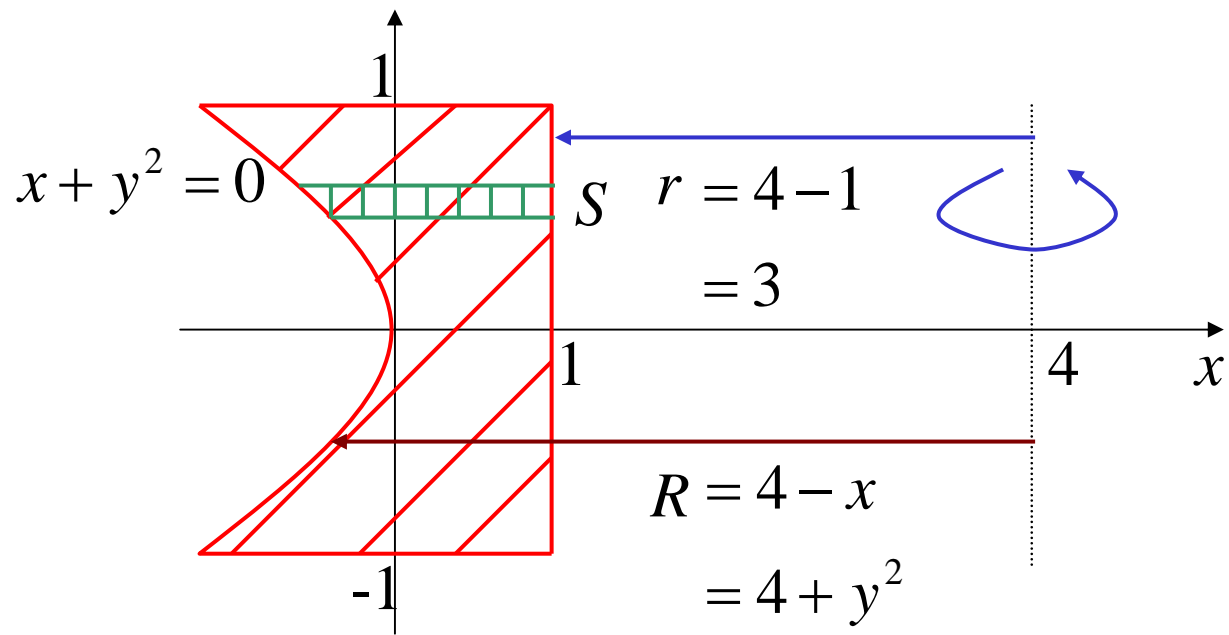
$$= \frac{3\pi}{10} \text{ units}^3$$

(iv) (1996)

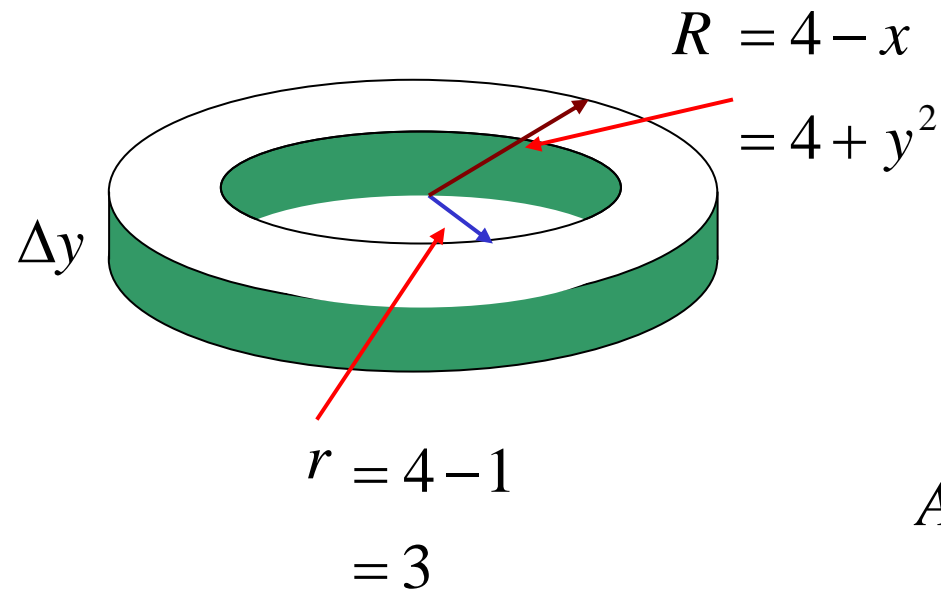


The shaded region is bounded by the lines $x = 1$, $y = 1$ and $y = -1$ and the curve $x + y^2 = 0$. The region is rotated through 360° about the line $x = 4$ to form a solid.

When the region is rotated, the line segment S at height y sweeps out an annulus.



a) Show that the area of the annulus at height y is equal to $\pi(y^4 + 8y^2 + 7)$



$$\begin{aligned} A(y) &= \pi \left\{ (4 + y^2)^2 - (3)^2 \right\} \\ &= \pi (16 + 8y^2 + y^4 - 9) \\ &= \pi (y^4 + 8y^2 + 7) \end{aligned}$$

b) Hence find the volume of the solid.

$$\Delta V = \pi(y^4 + 8y^2 + 7) \cdot \Delta y$$

$$V = \lim_{\Delta y \rightarrow 0} \sum_{y=-1}^1 \pi(y^4 + 8y^2 + 7) \cdot \Delta y$$

$$= 2\pi \int_0^1 (y^4 + 8y^2 + 7) dy$$

$$= 2\pi \left[\frac{1}{5} y^5 + \frac{8}{3} y^3 + 7y \right]_0^1$$

$$= 2\pi \left\{ \frac{1}{5} + \frac{8}{3} + 7 - 0 \right\}$$

$$= \frac{296\pi}{15} \text{ units}^3$$

Exercise 3A;
2, 5, 7, 9, 13, 14, 15

Exercise 3B;
1, 2, 5, 7, 9, 10, 11, 12