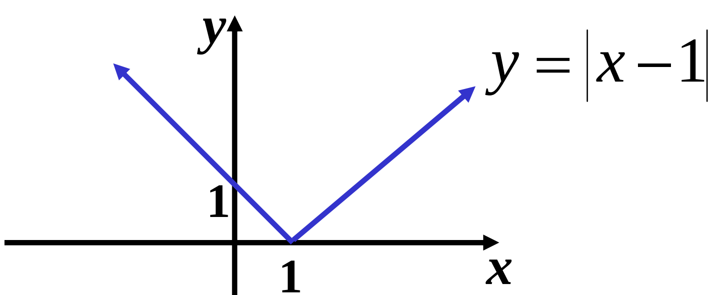
Limits & Continuity

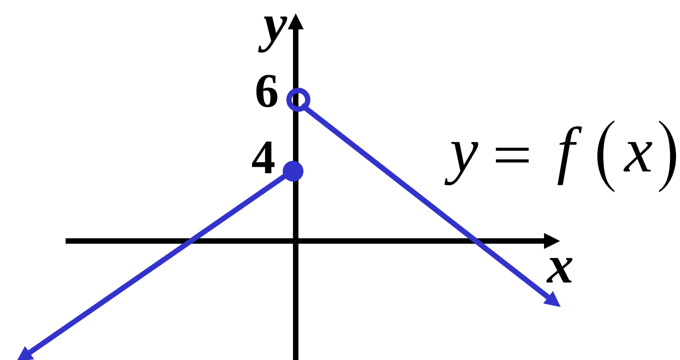
A limit describes the behaviour of functions.

$\lim_{x \to a} f(x)$: as the x value approaches ***a***, what value does $f(x)$ approach?

$\lim_{x \to a^-} f(x)$: as the x value approaches ***a*** from the negative side, what value does $f(x)$ approach?

$\lim_{x \to a^+} f(x)$: as the x value approaches ***a*** from the positive side, what value does $f(x)$ approach?





$\lim_{x \to 1^-} |x-1| = 0$ $\quad\lim_{x \to 1^+} |x-1| = 0$

$\lim_{x \to 0^-} f(x) = 4$ $\quad\lim_{x \to 0^+} f(x) = 6$

If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$, then $f(x)$ is continuous at $x = a$

Finding Limits

(1) Direct Substitution

e.g. $\lim_{x \to 5} x + 7 = 5 + 7$

$= 12$

(2) Factorise and Cancel

e.g. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} \frac{(x+3)(x-3)}{(x-3)}$

$= \lim_{x \to 3} (x+3)$

$= 3 + 7$

$= 6$

(3) **Special Limit**

$$\lim_{x\to\infty}\frac{1}{x}=0$$

e.g. (i) $\displaystyle\lim_{x\to\infty}\frac{x^3+3x^2+2x-1}{4x^3-1}=\lim_{x\to\infty}\frac{\frac{x^3}{x^3}+\frac{3x^2}{x^3}+\frac{2x}{x^3}-\frac{1}{x^3}}{\frac{4x^3}{x^3}-\frac{1}{x^3}}$

$$=\lim_{x\to\infty}\frac{1+\frac{3}{x}+\frac{2}{x^2}-\frac{1}{x^3}}{4-\frac{1}{x^3}}$$

$$=\frac{1}{4}$$

(ii) $\displaystyle\lim_{x\to\infty}\frac{4x-x^2}{x^3+1}=\frac{0}{1}$

$$=0$$

(iii) $\displaystyle\lim_{x\to\infty}\frac{x^7+x^6+x^2}{3x^7-x-974}=\frac{1}{3}$

$$(iv)\quad \lim_{x\to\infty}\frac{x^3+2}{x^2-1} = \frac{1}{0}$$

$$= \infty$$

(v) Find the horizontal asymptote of $y = \dfrac{(x+3)(x-2)}{(x-1)(x+1)}$

$$\lim_{x\to\infty}\frac{(x+3)(x-2)}{(x-1)(x+1)} = \lim_{x\to\infty}\frac{x^2+x-6}{x^2-1}$$

$$= \frac{1}{1}$$

$$= 1$$

$\therefore$ horizontal asymptote is $y = 1$

Exercise 7I; 1a, 2ace, 3ac, 4a, 5ad, 8a, 9ab, 10a