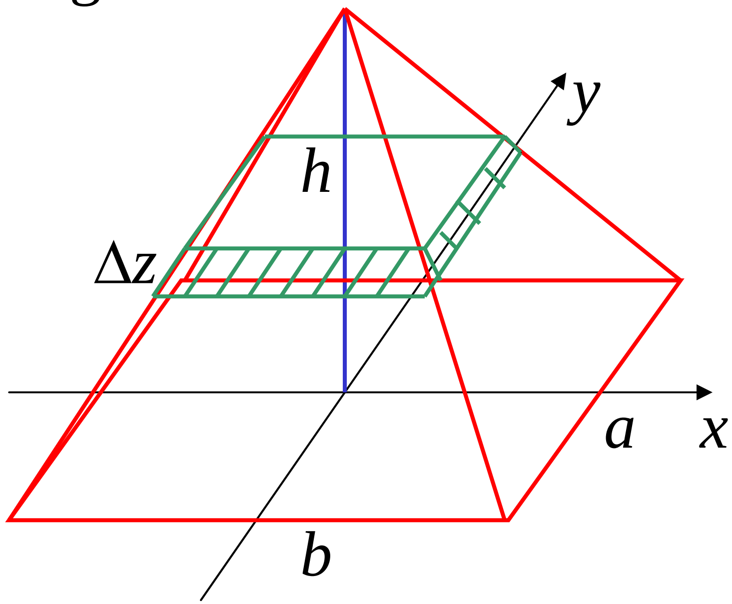
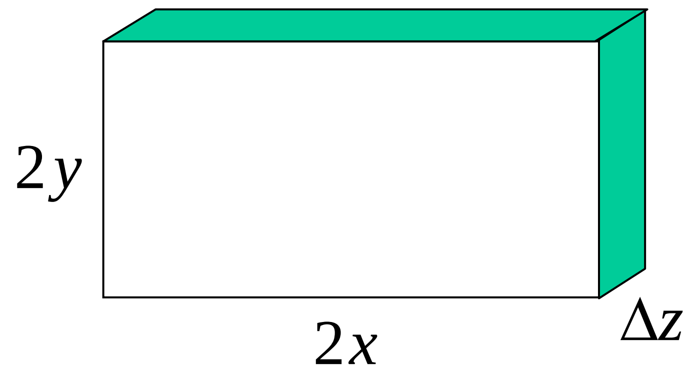
Volumes By Non Circular Cross-Sections

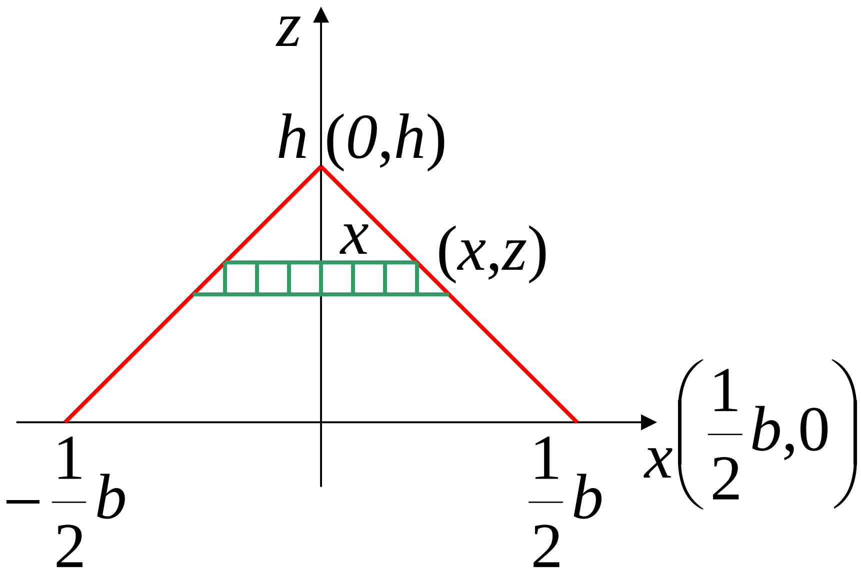
e.g. Find the volume of this rectangular pyramid



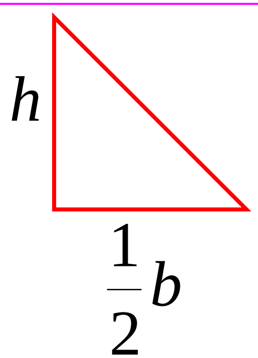


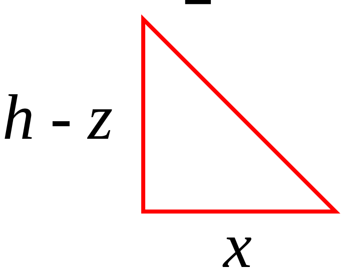
$$A(z) = 4xy$$

Find x in terms of z



Method 1: using similar Δ′s





$$\frac{h}{\frac{1}{2}b} = \frac{h-z}{x}$$

$$\frac{2h}{b} = \frac{h-z}{x}$$

$$x = \frac{b(h-z)}{2h}$$

Method 2: using coordinate geometry

$$m = \frac{h-0}{0-\frac{1}{2}b}$$

$$= \frac{-2h}{b}$$

$$z-h = \frac{-2h}{b}(x-0)$$

$$b(z-h) = -2hx$$

$$x = \frac{b(h-z)}{2h}$$

Similarly;

$$y = \frac{a(h-z)}{2h}$$

$$A(z) = 4\left[\frac{b(h-z)}{2h}\right]\left[\frac{a(h-z)}{2h}\right]$$

$$= \frac{ab(h-z)^2}{h^2}$$

$$\Delta V = \frac{ab(h-z)^2}{h^2} \cdot \Delta z$$

$$V = \lim_{\Delta z \to 0} \sum_{z=0}^{h} \frac{ab(h-z)^2}{h^2} \cdot \Delta z$$

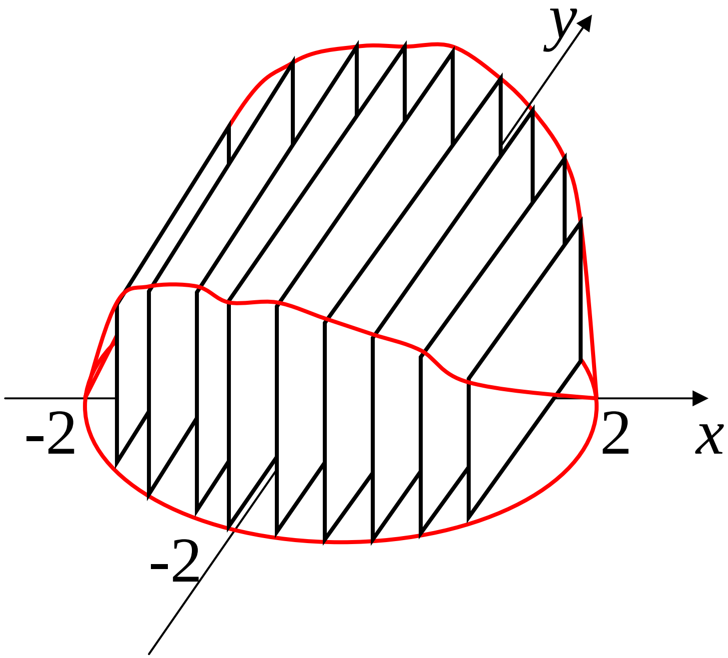
$$= \frac{ab}{h^2} \int_0^h (h-z)^2 \, dz$$

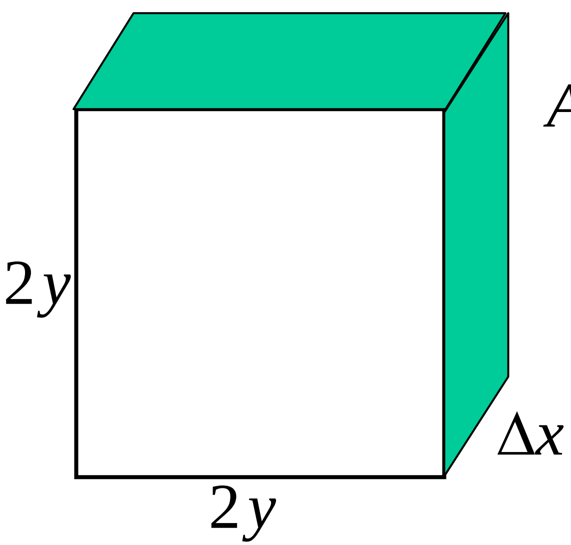
$$= \frac{ab}{h^2} \left[\frac{(h-z)^3}{-3}\right]_0^h$$

$$= \frac{ab}{h^2} \left\{0 + \frac{h^3}{3}\right\}$$

$$= \frac{abh}{3} \text{ units}^3$$

(ii) A solid has a circular base, $x^2 + y^2 = 4$, and each cross section is a square perpendicular to the base.





$$A(x) = 4y^2$$
$$= 4(4 - x^2)$$
$$\Delta V = 4(4 - x^2)\cdot \Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=-2}^{2} 4(4 - x^2)\cdot \Delta x$$
$$= 8\int_0^2 (4 - x^2)\,dx$$
$$= 8\left[4x - \frac{1}{3}x^3\right]_0^2$$
$$= 8\left(8 - \frac{8}{3} - 0\right)$$
$$= \frac{128}{3} \text{ units}^3$$

Exercise 3A; 16ade, 19

Exercise 3C; 2, 7, 8