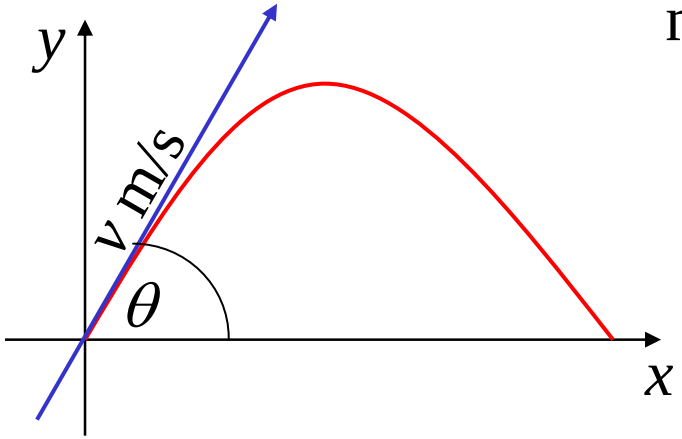
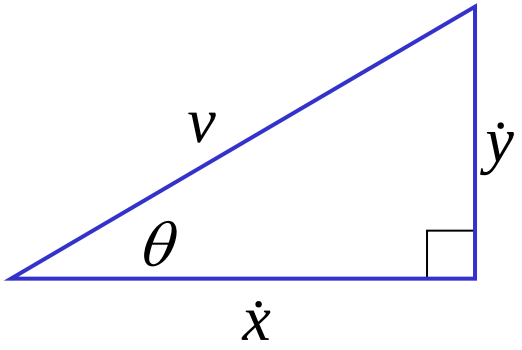
Projectile Motion



maximum range

$\theta = 45^\circ$

Initial conditions when $t = 0$



$$\frac{\dot{x}}{v} = \cos\theta \qquad \frac{\dot{y}}{v} = \sin\theta$$

$$\dot{x} = v\cos\theta \qquad \dot{y} = v\sin\theta$$

$$x = 0 \qquad y = 0$$

$$\ddot{x} = 0$$

$$\dot{x} = c_1$$

when $t = 0, \dot{x} = v\cos\theta$

$$c_1 = v\cos\theta$$

$$\dot{x} = v\cos\theta$$

$$x = vt\cos\theta + c_3$$

when $t = 0, x = 0$

$$c_3 = 0$$

$$x = vt\cos\theta$$

$$\ddot{y} = -g$$

$$\dot{y} = -gt + c_2$$

$$\dot{y} = v\sin\theta$$

$$c_2 = v\sin\theta$$

$$\dot{y} = -gt + v\sin\theta$$

$$y = -\frac{1}{2}gt^2 + vt\sin\theta + c_4$$

$$y = 0$$

$$c_4 = 0$$

$$y = -\frac{1}{2}gt^2 + vt\sin\theta$$

Common Questions

(1) When does the particle hit the ground?

Particle hits the ground when $y = 0$

(2) What is the range of the particle?

(i) find when $y = 0$

(ii) substitute into x

(3) What is the greatest height of the particle?

(i) find when $\dot{y} = 0$

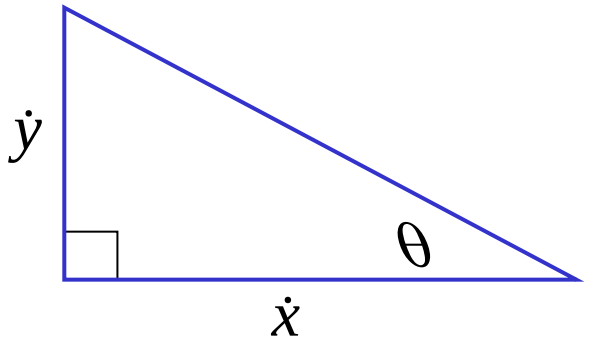
(ii) substitute into y

(4) What angle does the particle make with the ground?

(i) find when $y = 0$

(ii) substitute into $\dot{y}$

(iii) $\tan\theta = \dfrac{\dot{y}}{\dot{x}}$



Summary

A particle undergoing projectile motion obeys

$$\ddot{x} = 0 \qquad \text{and} \qquad \ddot{y} = -g$$

with initial conditions

$$\dot{x} = v\cos\theta \qquad \text{and} \qquad \dot{y} = v\sin\theta$$

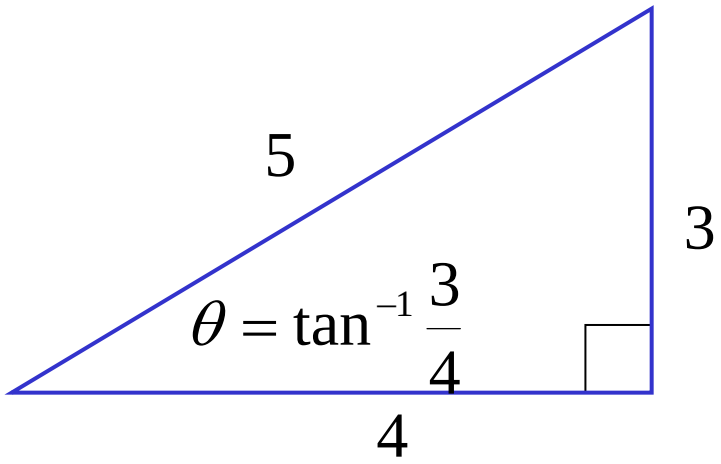
e.g. A ball is thrown with an initial velocity of 25 m/s at an angle of $\theta = \tan^{-1}\frac{3}{4}$ to the ground. Determine;.

a) greatest height obtained

Initial conditions $\dot{x} = v\cos\theta$ $\qquad \dot{y} = v\sin\theta$

$\dot{x} = 25\left(\frac{4}{5}\right)$ $\qquad \dot{y} = 25\left(\frac{3}{5}\right)$

$= 20\text{m/s}$ $\qquad = 15\text{m/s}$



$\ddot{x} = 0$

$\dot{x} = c_1$

when $t = 0, \dot{x} = 20$

$c_1 = 20$

$\dot{x} = 20$

$x = 20t + c_3$

when $t = 0, x = 0$

$c_3 = 0$

$x = 20t$

$\ddot{y} = -10$

$\dot{y} = -10t + c_2$

$\dot{y} = 15$

$c_2 = 15$

$\dot{y} = -10t + 15$

$y = -5t^2 + 15t + c_4$

$y = 0$

$c_4 = 0$

$y = -5t^2 + 15t$

greatest height occurs when $\dot{y} = 0$

$-10t + 15 = 0$

$t = \frac{3}{2}$

when $t = \frac{3}{2}, y = -5\left(\frac{3}{2}\right)^2 + 15\left(\frac{3}{2}\right)$

$= \frac{45}{4}$

$\therefore$ greatest height is $11\frac{1}{4}$ m above the ground

b) range

time of flight is 3 seconds

when $t = 3, x = 20(3)$

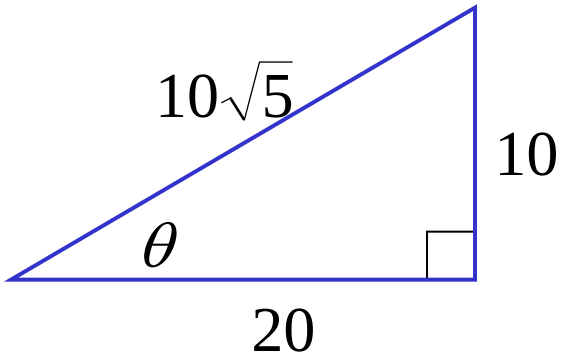
$= 60$

$\therefore$ range is 60m

c) velocity and direction of the ball after $\frac{1}{2}$ second

when $t = \frac{1}{2}, \dot{x} = 20$ $\quad \dot{y} = -10\left(\frac{1}{2}\right) + 15$

$= 10$



$\tan\theta = \frac{1}{2}$

$\theta = 26^\circ 34'$

$\therefore$ after $\frac{1}{2}$ second, velocity $= 10\sqrt{5}$m/s and it is traveling at an angle of $26^\circ 34'$ to the horizontal

d) cartesian equation of the path

$$x = 20t \qquad y = -5t^2 + 15t$$

$$t = \frac{x}{20} \qquad y = -5\left(\frac{x}{20}\right)^2 + 15\left(\frac{x}{20}\right)$$

$$y = \frac{-x^2}{80} + \frac{3x}{4}$$

Exercise 3G; 1ac, 2ac, 4, 6, 8, 9, 11, 13, 16, 18

Exercise 3H; 2, 4, 6, 7, 10, 11