

Double Angles

$$\sin 2\theta = \sin(\theta + \theta)$$

$$= \sin \theta \cos \theta + \cos \theta \sin \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos(\theta + \theta)$$

$$= \cos \theta \cos \theta - \sin \theta \sin \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \cos^2 \theta - (1 - \cos^2 \theta)$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$= 2(1 - \sin^2 \theta) - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\begin{aligned} \tan 2\theta &= \tan(\theta + \theta) \\ &= \frac{\tan \theta + \tan \theta}{1 - \tan \theta \tan \theta} \end{aligned}$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Double Angles

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1 \quad \Rightarrow \cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

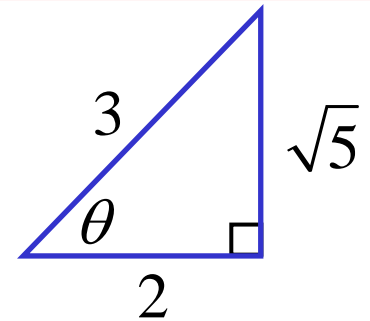
$$= 1 - 2 \sin^2 \theta \quad \Rightarrow \sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

e.g. (i) If $\cos \theta = \frac{2}{3}$, find $\tan 2\theta$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\begin{aligned} \tan 2\theta &= \frac{2 \left(\frac{\sqrt{5}}{2} \right)}{1 - \left(\frac{\sqrt{5}}{2} \right)^2} \\ &= \frac{\sqrt{5}}{-\frac{1}{4}} \\ &= \underline{\underline{-4\sqrt{5}}} \end{aligned}$$



(ii) Find the exact value of $\sin \frac{5\pi}{12} \cos \frac{5\pi}{12}$

$$\sin \frac{5\pi}{12} \cos \frac{5\pi}{12} = \frac{1}{2} \left(2 \sin \frac{5\pi}{12} \cos \frac{5\pi}{12} \right)$$

$$= \frac{1}{2} \sin \left(2 \times \frac{5\pi}{12} \right)$$

$$= \frac{1}{2} \sin \frac{5\pi}{6}$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \underline{\underline{\frac{1}{4}}}$$

(iii) If $\cos \theta = \frac{2}{3}$, find the exact value of $\sin \frac{\theta}{2}$

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$\therefore \sin^2 \frac{\theta}{2} = \frac{1}{2}(1 - \cos \theta)$$

$$= \frac{1}{2} \left(1 - \frac{2}{3} \right)$$

$$= \frac{1}{6}$$

$$\underline{\sin \frac{\theta}{2} = \pm \frac{1}{\sqrt{6}}}$$

(iv) Prove $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} \equiv \tan x$

$$\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \sqrt{\frac{1 - (1 - 2\sin^2 x)}{1 + (2\cos^2 x - 1)}}$$

$$= \sqrt{\frac{2\sin^2 x}{2\cos^2 x}}$$

$$= \sqrt{\frac{\sin^2 x}{\cos^2 x}}$$

$$= \sqrt{\tan^2 x}$$

$$= \underline{\tan x}$$

(v) Prove that $\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} = 2$ 1996 Extension 1 HSC Q4a)

$$\begin{aligned}\frac{\sin 3\theta}{\sin \theta} - \frac{\cos 3\theta}{\cos \theta} &= \frac{\sin 3\theta \cos \theta - \cos 3\theta \sin \theta}{\sin \theta \cos \theta} \\ &= \frac{2 \sin(3\theta - \theta)}{2 \sin \theta \cos \theta} \\ &= \frac{2 \sin 2\theta}{\sin 2\theta} \\ &= \underline{2}\end{aligned}$$

(vi) Prove the following identity;

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$$\frac{2 \tan A}{1 + \tan^2 A} = \sin 2A$$

$$\begin{aligned} \frac{2 \tan A}{1 + \tan^2 A} &= \frac{2 \sin A}{\cos A} \\ &= \frac{2 \sin A}{1 + \frac{\sin^2 A}{\cos^2 A}} \\ &= \frac{2 \sin A \cos A}{\cos^2 A + \sin^2 A} \\ &= \frac{\sin 2A}{1} \\ &= \underline{\sin 2A} \end{aligned}$$

Book2

Exercise 2A; 2ade, 3bde, 5adej, 7, 8adg, 10ab, 11, 13ck, 16, 19*