

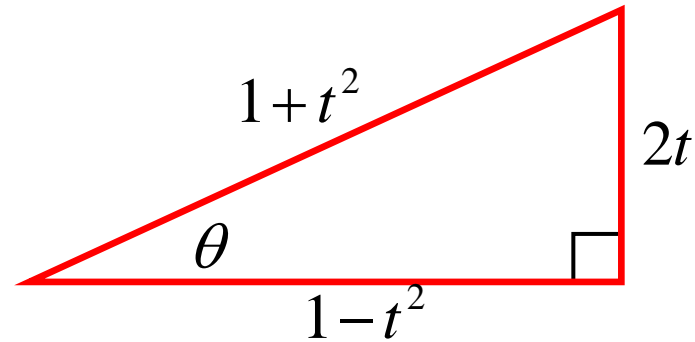
The t results

$$\text{Let } t = \tan \frac{\theta}{2}$$

$$\tan \theta = \frac{2 \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}}$$

$$\tan \theta = \frac{2t}{1-t^2}$$

$$\text{If } t = \tan \frac{\theta}{2};$$



$$\begin{aligned} h^2 &= (2t)^2 + (1-t^2)^2 \\ &= 4t^2 + 1 - 2t^2 + t^4 \\ &= t^4 + 2t^2 + 1 \\ &= (t^2 + 1)^2 \end{aligned}$$

$$\tan \theta = \frac{2t}{1-t^2} \quad \sin \theta = \frac{2t}{1+t^2} \quad \cos \theta = \frac{1-t^2}{1+t^2}$$

Note:

$$\text{If } t = \tan \theta; \quad \tan 2\theta = \frac{2t}{1-t^2}$$

$$\text{If } t = \tan 2\theta; \quad \tan 4\theta = \frac{2t}{1-t^2}$$

e.g. (i) Show that $\frac{1 - \cos x}{\sin x} = t$, where $t = \tan \frac{x}{2}$

$$\frac{1 - \cos x}{\sin x} = \frac{1 - \frac{1 - t^2}{1 + t^2}}{\frac{2t}{1 + t^2}} \left(\begin{array}{l} x \text{ is double } \frac{x}{2}, \\ \text{so } t \text{ results can be used for } \sin x, \cos x \end{array} \right)$$

$$= \frac{1 + t^2 - (1 - t^2)}{2t}$$

$$= \frac{2t^2}{2t}$$

$$= t$$

(ii) Use $t = \tan \frac{\theta}{2}$ to simplify $\frac{2 \tan 75^\circ}{1 + \tan^2 75^\circ}$

$$\frac{2 \tan 75^\circ}{1 + \tan^2 75^\circ} = \frac{2t}{1 + t^2}$$

$$= \sin \theta$$

$$= \sin 150^\circ$$

$$= \underline{\underline{\frac{1}{2}}}$$

Let $t = \tan 75^\circ$;

i.e. $\frac{\theta}{2} = 75^\circ$

$$\theta = 150^\circ$$

(iii) Prove $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan \frac{\theta}{2}$

Let $t = \tan \frac{\theta}{2}$;

$$\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \frac{1 + \frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{1 + \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}$$

$$= \frac{1+t^2 + 2t - (1-t^2)}{1+t^2 + 2t + 1-t^2}$$

$$= \frac{2t^2 + 2t}{2 + 2t}$$

$$= \frac{2t(t+1)}{2(1+t)} = t = \underline{\underline{\tan \frac{\theta}{2}}}$$

(iv) By making the substitution $t = \tan \frac{\theta}{2}$ or otherwise,

show that $\operatorname{cosec} \theta + \cot \theta = \cot \frac{\theta}{2}$

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$$\begin{aligned}\operatorname{cosec} \theta + \cot \theta &= \frac{1+t^2}{2t} + \frac{1-t^2}{2t} \\ &= \frac{2}{2t} \\ &= \frac{1}{t} \\ &= \cot \frac{\theta}{2}\end{aligned}$$

Exercise 2B; 1def, 3ace, 4bcf, 5aceg, 6, 8bd, 10a