Odd & Even Functions

(1) Even

$$f(-x) = f(x)$$

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

NOTE: horizontal shift $\displaystyle\int_{c-a}^{c+a} f(x-c)dx = 2\int_{c}^{c+a} f(x-c)dx$

(2) Odd

$$f(-x) = -f(x)$$

$$\int_{-a}^{a} f(x)dx = 0$$

NOTE: horizontal shift $\displaystyle\int_{c-a}^{c+a} f(x-c)dx = 0$

(3)

$$\int_{0}^{a} f(x)dx = \int_{0}^{a} f(a-x)dx$$

Proof:

$$\int_0^a f(a-x)\,dx$$

$$= -\int_a^0 f(u)\,du$$

$$= \int_0^a f(u)\,du$$

$$= \int_0^a f(x)\,dx$$

$$u = a - x$$

$$du = -dx$$

$$x = 0, u = a$$

$$x = a, u = 0$$

$$\text{odd} \times \text{odd} = \text{even}$$

$$\text{odd} \times \text{even} = \text{odd}$$

$$\text{even} \times \text{even} = \text{even}$$

e.g. $(i)\int_{-1}^{1}\sin^3 x\,dx = 0$ $\qquad (\text{odd function})^3 = \text{odd function}$

$$(ii)\int_{0}^{1} x^2\sqrt{1-x}\,dx = \int_{0}^{1}(1-x)^2\sqrt{x}\,dx$$

$$= \int_{0}^{1}\left(x^{\frac{1}{2}} - 2x^{\frac{3}{2}} + x^{\frac{5}{2}}\right)dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{4}{5}x^{\frac{5}{2}} + \frac{2}{7}x^{\frac{7}{2}}\right]_0^1$$

$$= \frac{2}{3} - \frac{4}{5} + \frac{2}{7} - 0$$

$$= \frac{16}{105}$$

Exercise 2I; 1 bdf, 2 ace, 3

Exercise 2J; 42, 44

The 100 ***(not 78)***