

Products to Sums

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

$$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$$

$$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$$

eg (i) Express as a sum or difference of trig functions

a) $2 \cos 5x \sin x = 2 \sin x \cos 5x$

$$= \sin(x + 5x) + \sin(x - 5x)$$

$$= \sin 6x + \sin(-4x)$$

$$= \underline{\sin 6x - \sin 4x}$$

b) $\cos 3\theta \cos 5\theta = \frac{1}{2}(2 \cos 3\theta \cos 5\theta)$

$$= \frac{1}{2}(\cos(3\theta + 5\theta) + \cos(3\theta - 5\theta))$$

$$= \frac{1}{2}(\cos 8\theta + \cos(-2\theta)) = \underline{\underline{\frac{1}{2}(\cos 8\theta + \cos 2\theta)}}$$

$$\begin{aligned}
 \text{(ii) Evaluate } 2 \sin 45^\circ \cos 15^\circ &= \sin(45 + 15) + \sin(45 - 15) \\
 &= \sin 60^\circ + \sin 30^\circ \\
 &= \frac{\sqrt{3}}{2} + \frac{1}{2} \\
 &= \frac{\sqrt{3} + 1}{2}
 \end{aligned}$$

Sums to Products

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B) \quad (2 \text{ sine half sum cos half diff})$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B) \quad (2 \text{ cos half sum cos half diff})$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B) \quad (\text{minus } 2 \text{ sine half sum sine half diff})$$

eg (i) Convert into products of trig functions

$$\begin{aligned} a) \cos 3A - \cos 5A &= -2 \sin \frac{1}{2}(8A) \sin \frac{1}{2}(-2A) \\ &= -2 \sin 4A \sin(-A) \\ &= \underline{2 \sin 4A \sin A} \end{aligned}$$

$$\begin{aligned} b) \sin 6x - \sin 4x &= \sin 6x + \sin(-4x) \\ &= 2 \sin \frac{1}{2}(2x) \cos \frac{1}{2}(10x) \\ &= \underline{2 \sin x \cos 5x} \end{aligned}$$

(ii) Solve $\sin x + \sin 3x = 0$ $0^\circ \leq x \leq 360^\circ$

$$2 \sin 2x \cos(-x) = 0$$

$$2 \sin 2x \cos x = 0$$

$$\sin 2x = 0 \quad \text{or} \quad \cos x = 0$$

$$2x = 0^\circ, 180^\circ, 360^\circ, 540^\circ, 720^\circ$$

$$x = 90^\circ, 270^\circ$$

$$x = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$$

$$\therefore \underline{x = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ}$$

Exercise 2F; 1b, 2b, 3a, 9, 10ace