

Equations of the form $a\sin x + b\cos x = c$

Method 1: Using the t results

eg (i) $3\cos\theta + 4\sin\theta = 2$ $0 \leq \theta \leq 360^\circ$

let $t = \tan\frac{\theta}{2}$ $3\left(\frac{1-t^2}{1+t^2}\right) + 4\left(\frac{2t}{1+t^2}\right) = 2$ $0 \leq \frac{\theta}{2} \leq 180^\circ$

$$3 - 3t^2 + 8t = 2 + 2t^2$$

$$5t^2 - 8t - 1 = 0$$

$$t = \frac{8 \pm \sqrt{84}}{10}$$

$$\tan\frac{\theta}{2} = \frac{4 - \sqrt{21}}{5}$$

or

$$\tan\frac{\theta}{2} = \frac{4 + \sqrt{21}}{5}$$

Q2 $\tan\alpha = \frac{\sqrt{21} - 4}{5}$

Q1 $\tan\alpha = \frac{4 + \sqrt{21}}{5}$

$$\alpha = 6^\circ 39'$$

$$\frac{\theta}{2} = 173^\circ 21'$$

$$\theta = 346^\circ 42'$$

$$\alpha = 59^\circ 47'$$

$$\frac{\theta}{2} = 59^\circ 47'$$

$$\theta = 119^\circ 33'$$

Test: $\theta = 180^\circ$

$$3\cos 180^\circ + 4\sin 180^\circ = -4 \neq 2$$

$\therefore \theta = 119^\circ 33', 346^\circ 42'$

Method 2: Auxiliary Angle Method

(i) Change into a sine function

eg (i) $3\cos\theta + 4\sin\theta = 2$

$$0 \leq \theta \leq 360^\circ$$

$$3\cos\theta + 4\sin\theta = 2$$

$$5 \times \left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta \right) = 2$$

$$5\sin(\alpha + \theta) = 2$$

$$\sin(\alpha + \theta) = \frac{2}{5}$$

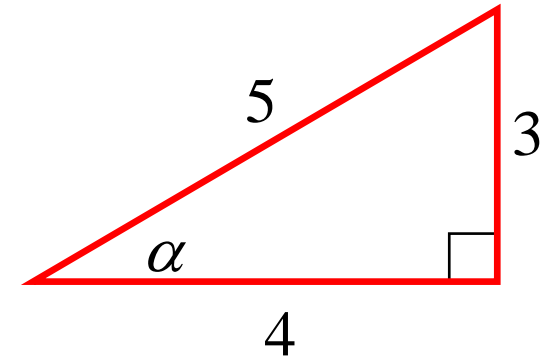
Q1, Q2

$$\sin\beta = \frac{2}{5}$$

$$\beta = 23^\circ 35'$$

$$36^\circ 52' + \theta = 23^\circ 35', 156^\circ 25'$$

$$\theta = -13^\circ 17', 119^\circ 33'$$



$$\tan\alpha = \frac{3}{4}$$

$$\alpha = 36^\circ 52'$$

$$\therefore \underline{\theta = 119^\circ 33', 346^\circ 43'}$$

Method 2: Auxiliary Angle Method

(ii) Change into a cosine function

eg (i) $3\cos\theta + 4\sin\theta = 2$ $0 \leq \theta \leq 360^\circ$

$$\cos\alpha \cos\theta + \sin\alpha \sin\theta$$

$$3\cos\theta + 4\sin\theta = 2$$

$$5 \times \left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta \right) = 2$$

$$5\cos(\theta - \alpha) = 2$$

$$\cos(\theta - \alpha) = \frac{2}{5}$$

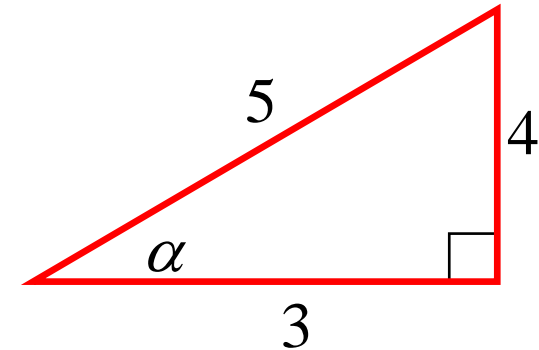
Q1, Q4

$$\cos\beta = \frac{2}{5}$$

$$\beta = 66^\circ 25'$$

$$\theta - 53^\circ 8' = 66^\circ 25', 293^\circ 35'$$

$$\therefore \theta = \underline{119^\circ 33', 346^\circ 43'}$$



$$\tan\alpha = \frac{4}{3}$$

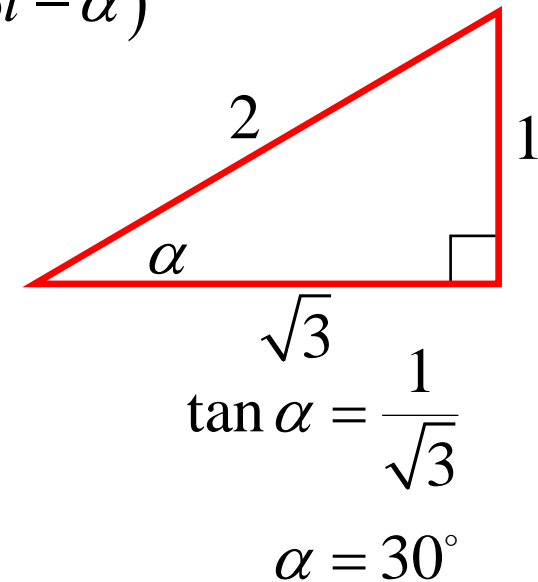
$$\alpha = 53^\circ 8'$$

2005 Extension 1 HSC Q5c) (i)

(ii) Express $\sqrt{3} \sin 3t - \cos 3t$ in the form $R \sin(3t - \alpha)$

$$(\sin 3t \cos \alpha - \cos 3t \sin \alpha)$$

$$\begin{aligned}\sqrt{3} \sin 3t - \cos 3t &= 2 \sin(3t - \alpha) \\ &= \underline{2 \sin(3t - 30^\circ)}\end{aligned}$$



Exercise 2E;

6, 7, 10bd, 11, 13, 14, 16ac, 20a, 23

2003 Extension 1 HSC Q2e) (i)

(iii) Express $\sin x - \cos x$ in the form $R \cos(x + \alpha)$

$$(\cos x \cos \alpha - \sin x \sin \alpha)$$

$$\begin{aligned}\sin x - \cos x &= -(\cos x - \sin x) \\ &= -\sqrt{2} \cos(x - \alpha) \\ &= \underline{-\sqrt{2} \cos(x - 45^\circ)}\end{aligned}$$

