The Quadratic Polynomial and the Parabola

Quadratic polynomial – $ax^2 + bx + c$

Quadratic function – $y = ax^2 + bx + c$

Quadratic equation – $ax^2 + bx + c = 0$

Coefficients – a, b, c

Indeterminate – x

Roots – Solutions to the quadratic equation

Zeroes – x intercepts of the quadratic function

e.g. Find the roots of $x^2 - 1 = 0$
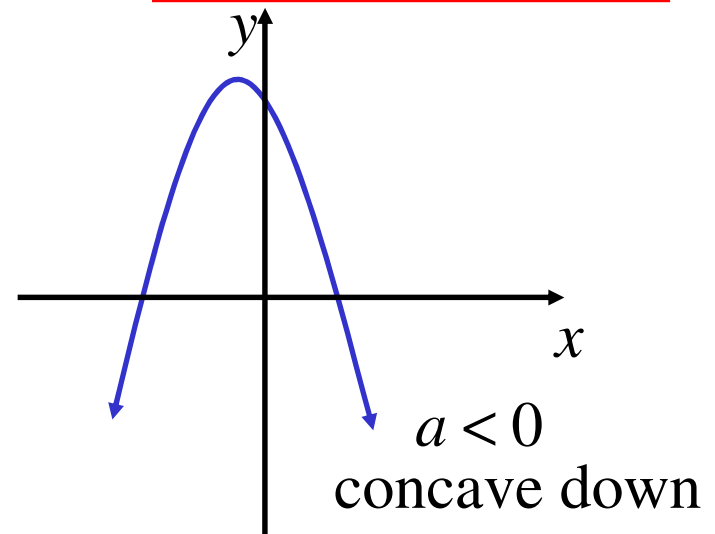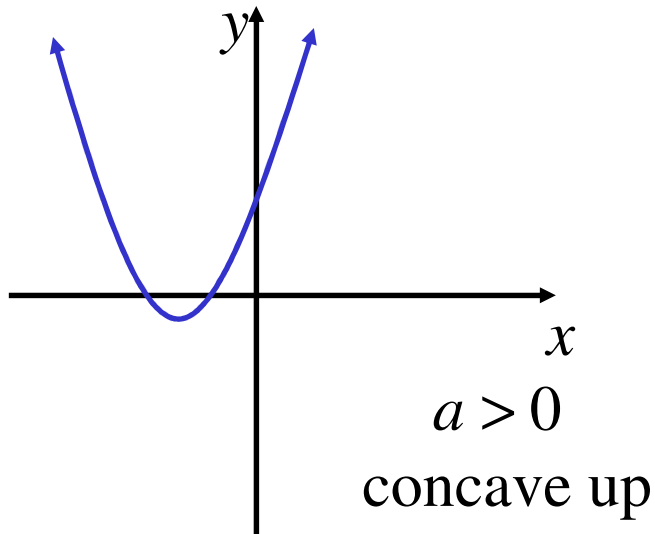
$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1 \qquad \therefore \text{ the roots are } x = -1 \text{ and } x = 1$$

Graphing Quadratics

The graph of a quadratic function is a parabola.

$$y = ax^2 + bx + c$$

a

$a > 0$
concave up

$a < 0$
concave down

c $= y$ intercept

zeroes (roots) $= x$ intercepts

$x = \dfrac{-b}{2a}$ $= $ axis of symmetry

Note: AOS is the average of the zeroes

vertex x value is the AOS

y value is found by substituting AOS into the function.
(It is the maximum/minimum value of the function)

e.g. Graph $y = x^2 + 8x + 12$

$\underline{\mathbf{a}} = 1 > 0 \therefore$ concave up

$\underline{\mathbf{zeroes}}$ $\quad x^2 + 8x + 12 = 0$

$\qquad (x+6)(x+2) = 0$

$\qquad x = -6 \text{ or } x = -2$

$\qquad \therefore \; x \text{ intercepts are}$

$\qquad (-6, 0) \text{ and } (-2, 0)$

$\underline{\mathbf{AOS}} \quad x = \dfrac{-b}{2a} \quad \boldsymbol{OR} \quad x = \dfrac{-6-2}{2}$

$\qquad \quad = \dfrac{-8}{2} \qquad\qquad = -4$

$\qquad \quad = -4$

$\underline{\mathbf{vertex}} \quad y = (-4)^2 + 8(-4) + 12$

$\qquad\qquad = -4$

$\qquad \therefore \text{ vertex is } (-4, -4)$

$\underline{\mathbf{c}} = 12 \therefore \; y \text{ intercept is } (0, 12)$

$y = x^2 + 8x + 12$
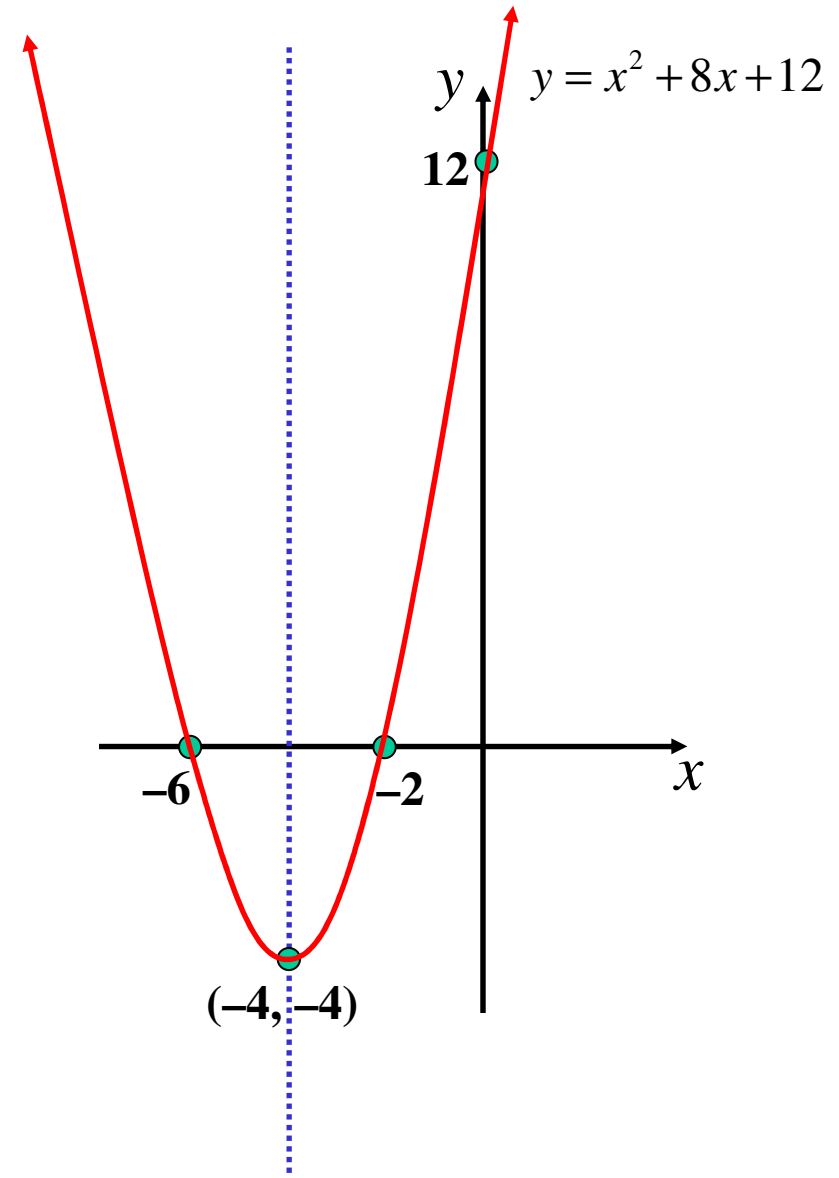
(*ii*) Find the quadratic with;

a) roots 3 and 6

$$y = a\left(x^2 - 9x + 18\right)$$

$-(6+3)$ 6×3

b) monic roots $3 + \sqrt{2}$ and $3 - \sqrt{2}$

$$y = x^2 - 6x + 7$$

$-\left(3 + \sqrt{2} + 3 - \sqrt{2}\right)$ $\left(3 + \sqrt{2}\right)\left(3 - \sqrt{2}\right)$

c) roots 2 and 8 and vertex (5,3)

$$y = a\left(x^2 - 10x + 16\right)$$

$$(5,3): \quad 3 = a\left(5^2 - 10(5) + 16\right)$$

$$3 = -9a$$

$$a = -\frac{1}{3}$$

$$\therefore y = -\frac{1}{3}\left(x^2 - 10x + 16\right)$$

(*iii*) Solve;

a) $x^2 + 5x + 6 > 0$

$(x+2)(x+3) > 0$

$x < -3$ or $x > -2$
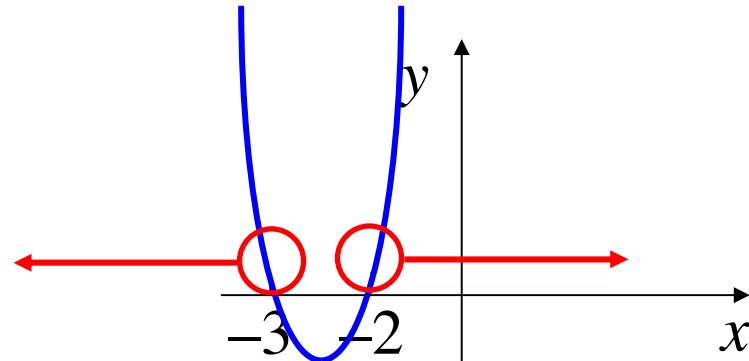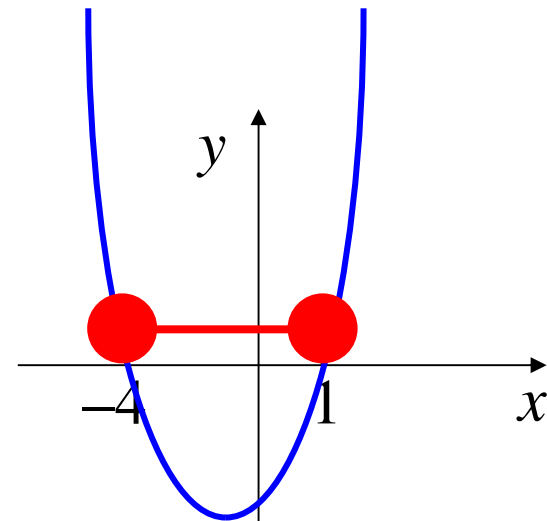
Q: for what values of x is the parabola above the x axis?

b) $-x^2 - 3x \geq -4$

$x^2 + 3x - 4 \leq 0$

$(x+4)(x-1) \leq 0$

$-4 \leq x \leq 1$

Q: for what values of x is the parabola below the x axis?

Exercise 8A; 1adf, 2adf, 3bd, 4bd, 5c, 6ade, 7d, 9ace, 12c, 13b, 14a