

General Expansion of Binomials

${}^n C_k$ is the coefficient of x^k in $(1+x)^k$

$${}^n C_k = \binom{n}{k}$$

$$(1+x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_n x^n$$

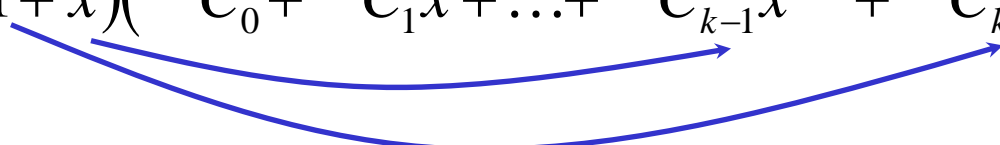
which extends to;

$$(a+b)^n = {}^n C_0 a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_{n-1} a b^{n-1} + {}^n C_n b^n$$

$$\begin{aligned} \text{e.g. } (2+3x)^4 &= {}^4 C_0 2^4 + {}^4 C_1 2^3 (3x) + {}^4 C_2 2^2 (3x)^2 + {}^4 C_3 2 (3x)^3 + {}^4 C_4 (3x)^4 \\ &= \underline{16 + 96x + 216x^2 + 216x^3 + 81x^4} \end{aligned}$$

Pascal's Triangle Relationships

$$(1) \quad {}^n C_k = {}^{n-1} C_{k-1} + {}^{n-1} C_k \quad \text{where } 1 \leq k \leq n-1$$

$$\begin{aligned} (1+x)^n &= (1+x)(1+x)^{n-1} \\ &= (1+x) \left({}^{n-1} C_0 + {}^{n-1} C_1 x + \dots + {}^{n-1} C_{k-1} x^{k-1} + {}^{n-1} C_k x^k + \dots + {}^{n-1} C_{n-1} x^{n-1} \right) \end{aligned}$$


looking at coefficients of x^k

$$\begin{aligned} LHS &= {}^n C_k & RHS &= (1)({}^{n-1} C_{k-1}) + (1)({}^{n-1} C_k) \\ & & &= {}^{n-1} C_{k-1} + {}^{n-1} C_k & \underline{\therefore {}^n C_k = {}^{n-1} C_{k-1} + {}^{n-1} C_k} \end{aligned}$$

$$(2) \quad {}^n C_k = {}^n C_{n-k} \quad \text{where } 1 \leq k \leq n-1$$

"Pascal's triangle is symmetrical"

$$(3) \quad {}^n C_0 = {}^n C_n = 1$$

**Exercise 5B; 2ace, 5, 6ac,
10ac, 11, 14**