General Expansion of Binomials

${}^nC_k$ is the coefficient of x^k in $(1+x)^k$

$${}^nC_k = \binom{n}{k}$$

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_n x^n$$

which extends to;

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n$$

$$e.g.(2+3x)^4 = {}^4C_0 2^4 + {}^4C_1 2^3 (3x) + {}^4C_2 2^2 (3x)^2 + {}^4C_3 2 (3x)^3 + {}^4C_4 (3x)^4$$

$$= 16 + 96x + 216x^2 + 216x^3 + 81x^4$$

Pascal's Triangle Relationships

(1) ${}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k} \quad \text{where } 1 \leq k \leq n-1$

$$(1+x)^{n} = (1+x)(1+x)^{n-1}$$
$$= (1+x)\left({}^{n-1}C_{0} + {}^{n-1}C_{1}x + \ldots + {}^{n-1}C_{k-1}x^{k-1} + {}^{n-1}C_{k}x^{k} + \ldots + {}^{n-1}C_{n-1}x^{n-1}\right)$$

looking at coefficients of x^{k}

$LHS = {}^{n}C_{k}$

$$RHS = (1)\left({}^{n-1}C_{k-1}\right) + (1)\left({}^{n-1}C_{k}\right)$$
$$= {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$$

$\therefore {}^{n}C_{k} = {}^{n-1}C_{k-1} + {}^{n-1}C_{k}$

(2) ${}^{n}C_{k} = {}^{n}C_{n-k} \quad \text{where } 1 \leq k \leq n-1$

"Pascal's triangle is symmetrical"

(3) ${}^{n}C_{0} = {}^{n}C_{n} = 1$

Exercise 5B; 2ace, 5, 6ac, 10ac, 11, 14