

Rules For Differentiation

(1) $y = c$

$$f(x) = c$$

$$f(x+h) = c$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{c - c}{h}$$

$$= \lim_{h \rightarrow 0} \frac{0}{h}$$

$$= \lim_{h \rightarrow 0} 0$$

$$= \underline{0}$$

(2) $y = kx$

$$f(x) = kx$$

$$f(x+h) = k(x+h)$$

$$= kx + kh$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{kx + kh - kx}{h}$$

$$= \lim_{h \rightarrow 0} \frac{kh}{h}$$

$$= \lim_{h \rightarrow 0} k$$

$$= \underline{k}$$

$$(3) \quad \underline{y = x^n}$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^4 - b^4 = (a - b)(a^3 + a^2b + ab^2 + b^3)$$

⋮

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + a^2b^{n-3} + ab^{n-2} + b^{n-1})$$

$$f(x) = x^n$$

$$f(x+h) = (x+h)^n$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h-x) \left\{ (x+h)^{n-1} + (x+h)^{n-2}x + \dots + (x+h)x^{n-2} + x^{n-1} \right\}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h \left\{ (x+h)^{n-1} + (x+h)^{n-2}x + \dots + (x+h)x^{n-2} + x^{n-1} \right\}}{h}$$

$$= \lim_{h \rightarrow 0} (x+h)^{n-1} + (x+h)^{n-2}x + \dots + (x+h)x^{n-2} + x^{n-1}$$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} (x+h)^{n-1} + (x+h)^{n-2}x + \dots + (x+h)x^{n-2} + x^{n-1} \\
 &= x^{n-1} + x^{n-1} + \dots + x^{n-1} + x^{n-1} \\
 &= \underline{nx^{n-1}}
 \end{aligned}$$

$$(4) \quad \underline{y = \frac{1}{x}}$$

$$f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{x - x - h}{hx(x+h)} \\
 &= \lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\
 &= \underline{\frac{-1}{x^2}}
 \end{aligned}$$

Note:

$$\begin{aligned}
 f(x) &= x^{-1} \\
 f'(x) &= -x^{-2} \\
 &= \frac{-1}{x^2}
 \end{aligned}$$

$$(5) \quad \underline{y = \sqrt{x}}$$

$$f(x) = \sqrt{x}$$

$$\begin{aligned} f(x+h) &= \sqrt{x+h} & f'(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ & & &= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} \\ & & &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\ & & &= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h} + \sqrt{x})} \\ & & &= \frac{1}{\sqrt{x} + \sqrt{x}} \\ & & &= \underline{\underline{\frac{1}{2\sqrt{x}}}} \end{aligned}$$

Note:

$$\begin{aligned} f(x) &= x^{\frac{1}{2}} \\ f'(x) &= \frac{1}{2} x^{-\frac{1}{2}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$

e.g. (i) $y = 7$
 $\frac{dy}{dx} = 0$

(ii) $y = 37x$
 $\frac{dy}{dx} = 37$

(iii) $y = x^{10}$
 $\frac{dy}{dx} = 10x^9$

(iv) $y = 3x^2 + 6x + 2$
 $\frac{dy}{dx} = 6x + 6$

(v) $y = (2x + 1)^2$
 $= 4x^2 + 4x + 1$
 $\frac{dy}{dx} = 8x + 4$

(vi) $y = 3x + \frac{1}{x^2}$
 $= 3x + x^{-2}$
 $\frac{dy}{dx} = 3 - 2x^{-3}$
 $= 3 - \frac{2}{x^3}$

(vii) $y = x^2 \sqrt{x}$
 $= x^{\frac{5}{2}}$
 $\frac{dy}{dx} = \frac{5}{2} x^{\frac{3}{2}}$
 $= \frac{5}{2} x \sqrt{x}$

(viii) If $f(x) = x^3 - 3$,
find $f'(2)$

$$f(x) = x^3 - 3$$

$$f'(x) = 3x^2$$

$$f'(2) = 3(2)^2$$

$$= 12$$

(xix) Find the equation of the tangent to the curve $y = 5x^3 - 6x^2 + 2$ at the point $(1,1)$

$$y = 5x^3 - 6x^2 + 2$$

$$\frac{dy}{dx} = 15x^2 - 12x$$

$$\text{when } x = 1, \frac{dy}{dx} = 15(1)^2 - 12(1) \\ = 3$$

\therefore required slope = 3

$$y - 1 = 3(x - 1)$$

$$y - 1 = 3x - 3$$

$$\underline{3x - y - 2 = 0}$$

(x) Find the points on the curve $y = x^3 - 12x$ where the tangents are horizontal

$$y = x^3 - 12x$$

$$\frac{dy}{dx} = 3x^2 - 12$$

tangents are horizontal when $\frac{dy}{dx} = 0$

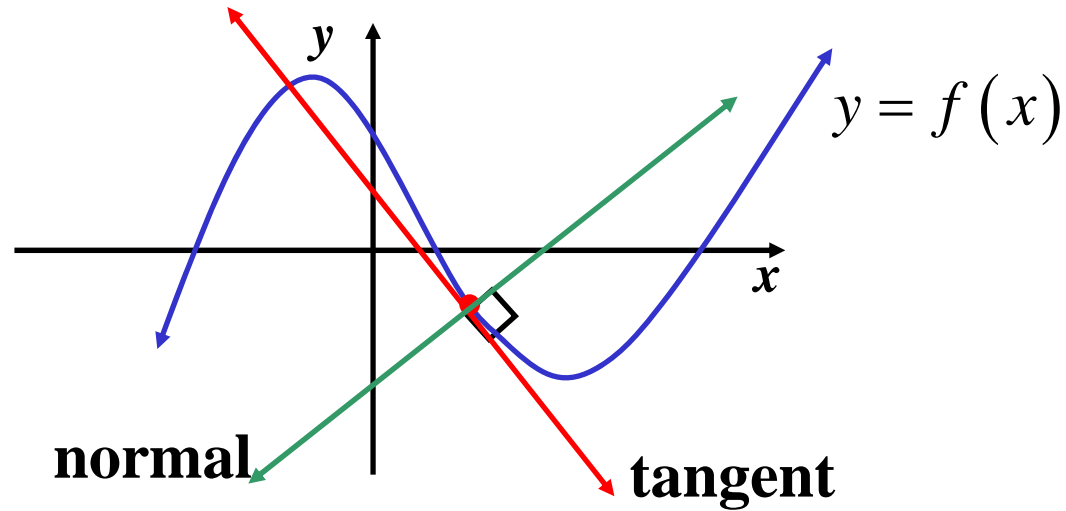
$$\text{i.e. } 3x^2 - 12 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

\therefore tangents are horizontal at $(-2,16)$ and $(2,-16)$

A **normal** is a line perpendicular to the tangent at the point of contact



(xi) Find the equation of the normal to the curve $y = 4x^2 - 3x + 2$ at the point $(3, 29)$

$$y = 4x^2 - 3x + 2$$

$$\frac{dy}{dx} = 8x - 3$$

$$\text{when } x = 3, \frac{dy}{dx} = 8(3) - 3$$
$$= 21$$

$$\therefore \text{required slope} = -\frac{1}{21}$$

$$y - 29 = -\frac{1}{21}(x - 3)$$

$$21y - 609 = -x + 3$$

$$\underline{x + 21y - 612 = 0}$$

**Exercise 7C; 1ace etc, 2ace etc, 3ace etc, 4bd, 5bdfh,
8bd, 9bd, 10ac, 12, 13b, 16, 21**

Exercise 7D; 2ac, 3bd, 4ace, 6c, 7b, 11a, 13aei, 18, 22