The Discriminant

$$\Delta = b^2 - 4ac$$

The discriminant tells us whether the roots are rational or irrational

$\Delta > 0$: two different real roots (cuts the x axis twice)

$\Delta = 0$: two equal real roots (touches the x axis once)

$\Delta < 0$: no real roots (never touches the x axis)

Δ is a perfect square : roots are rational

e.g. (i) Describe the roots of;

$a)\ 3x^2 + 5x + 9 = 0$

$\Delta = 5^2 - 4(3)(9)$

$= -83 < 0$

$\therefore$ no real roots

$b)\ 2x^2 + 6x - 3 = 0$

$\Delta = 6^2 - 4(2)(-3)$

$= 60 > 0$

$\therefore$ two different, real, irrational roots

(*ii*) Find the values of k which makes;

a) $x^2 + 6x + k = 0$ have equal roots

equal roots occur when $\Delta = 0$

i.e. $6^2 - 4k = 0$

$36 - 4k = 0$

$k = 9$

b) $x^2 - 4x + 2k = 0$ have unreal roots

unreal roots occur when $\Delta < 0$

i.e. $(-4)^2 - 4(2k) < 0$

$16 - 8k < 0$

$k > 2$

c) $kx^2 + 2x + 4k = 0$ have real roots

real roots occur when $\Delta \geq 0$

i.e. $2^2 - 4(k)(4k) \geq 0$

$$4 - 16k^2 \geq 0$$

$$k^2 \leq \frac{1}{4}$$

$$-\frac{1}{2} \leq k \leq \frac{1}{2}$$

(*iii*) For what value of a is the line $y = ax$ a tangent to the circle $x^2 + y^2 + 20x - 10y + 100 = 0$?

$$x^2 + a^2x^2 + 20x - 10ax + 100 = 0$$

$$(a^2 + 1)x^2 + 10(2 - a)x + 100 = 0$$

line is a tangent when $\Delta = 0$

$$\text{i.e. } 100(2-a)^2 - 4(a^2+1)(100) = 0$$

$$400 - 400a + 100a^2 - 400a^2 - 400 = 0$$

$$3a^2 + 4a = 0$$

$$a(3a+4) = 0$$

$$a = 0 \quad \text{or} \quad a = -\frac{4}{3}$$

Exercise 8F; 1ace, 2bdf, 3bg, 4ch, 5ad, 6, 7ac, 8be, 9ac, 11, 12b, 13, 14, 18, 21bd