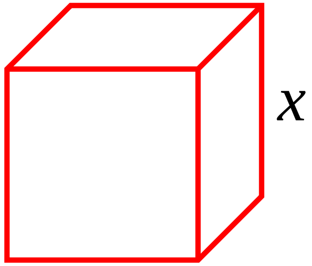
Rates of Change

e.g. A block of ice in the form of a cube has one edge 10 cm long. It is melting so that its dimensions decrease at the rate of 1 mm/s.

At what rate is the volume decreasing when the edge is 5cm long?



$$\frac{dV}{dt} = ? \qquad \frac{dx}{dt} = -\frac{1}{10} \qquad V = x^3$$

$$\frac{dV}{dx} = 3x^2$$

$$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$$

$$= 3x^2 \times -\frac{1}{10}$$

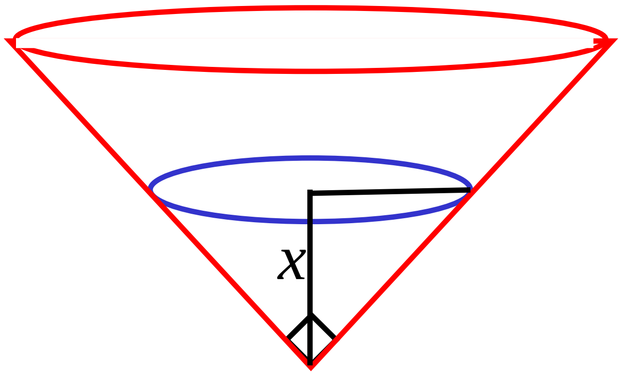
$$= -\frac{3x^2}{10}$$

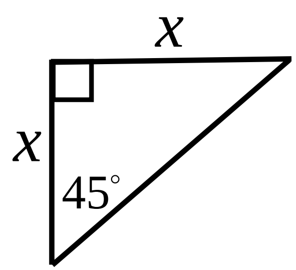
$$\text{when } x = 5, \frac{dV}{dt} = -\frac{3(5)^2}{10}$$

$$= -7.5$$

$\therefore$ volume is decreasing at 7.5 cm^3/s

(ii) A vessel is in the form of an inverted cone with a vertical angle of 90°
If the depth of the water in the vessel is x cm;
a) find the volume of water.





$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi x^2 x$$
$$= \frac{1}{3}\pi x^3$$

b) If water is poured in at a rate of 0.2 cm^3/min, find the rate the depth is increasing when the water depth is 4 cm.

$$\frac{dx}{dt} = ?$$
$$\frac{dV}{dt} = \frac{1}{5}$$
$$V = \frac{1}{3}\pi x^3$$
$$\frac{dV}{dx} = \pi x^2$$

$$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt}$$
$$= \frac{1}{\pi x^2} \times \frac{1}{5}$$
$$= \frac{1}{5\pi x^2}$$

when $x = 4$, $\frac{dx}{dt} = \frac{1}{5\pi(4)^2}$
$$= \frac{1}{80\pi}$$

$\therefore$ depth is increasing at $\frac{1}{80\pi}$ cm/min

Exercise 7H; 1a, 2a, 4, 6, 7, 8, 10, 12, 14