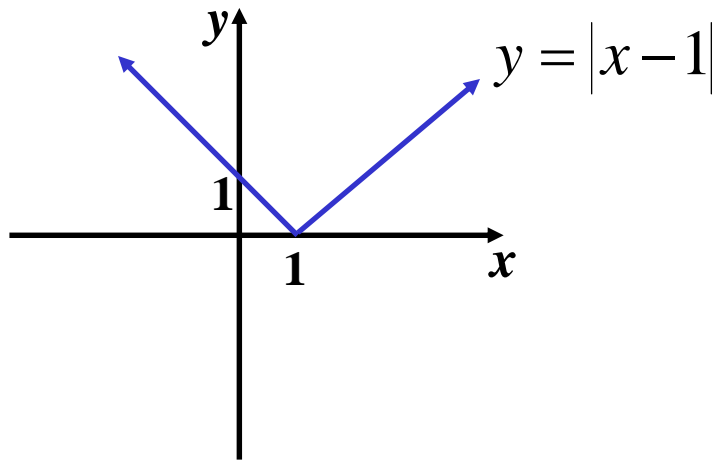


Differentiability

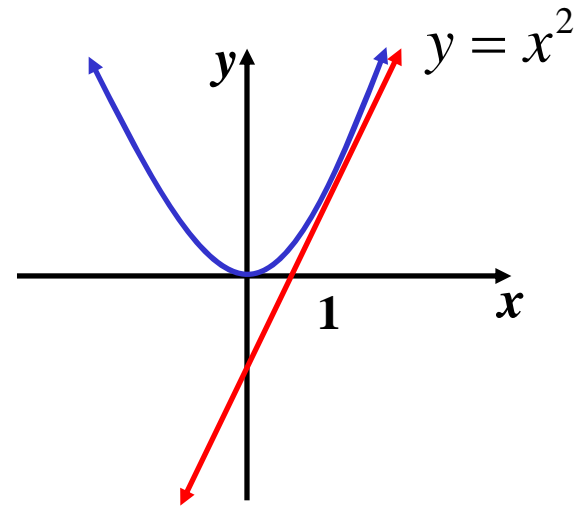
A function is differentiable at a point if the curve is **smooth continuous**

$$\text{i.e. } \lim_{x \rightarrow a^-} f'(x) = \lim_{x \rightarrow a^+} f'(x)$$



not differentiable at $x = 1$

$$\lim_{x \rightarrow 1^-} f'(x) = -1 \quad \lim_{x \rightarrow 1^+} f'(x) = 1$$



differentiable at $x = 1$

Implicit Differentiation

$$\frac{df}{dx} = \frac{df}{dy} \times \frac{dy}{dx}$$

e.g. (i) $x = y^2$

$$\frac{d}{dx}(x) = \frac{d}{dx}(y^2)$$

$$1 = 2y \frac{dy}{dx}$$

$$\underline{\frac{dy}{dx} = \frac{1}{2y}}$$

(ii) $\frac{d}{dx}(x^2 y^3)$

$$= (x^2) \left(3y^2 \frac{dy}{dx} \right) + (y^3)(2x)$$

$$= \underline{3x^2 y^2 \frac{dy}{dx} + 2xy^3}$$

$$\frac{d}{dx}(y^2) = \frac{d}{dy}(y^2) \times \frac{dy}{dx}$$

(iii) Find the equation of the tangent to $x^2 + y^2 = 9$ at the point $(1, 2\sqrt{2})$

$$x^2 + y^2 = 9$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\text{at } (1, 2\sqrt{2}), \frac{dy}{dx} = -\frac{1}{2\sqrt{2}}$$

$$\therefore \text{required slope} = -\frac{1}{2\sqrt{2}}$$

$$y - 2\sqrt{2} = -\frac{1}{2\sqrt{2}}(x - 1)$$

$$2\sqrt{2}y - 8 = -x + 1$$

$$\underline{x + 2\sqrt{2}y - 9 = 0}$$

**Exercise 7K; 1acegi, 2bdfh, 3a,
4a, 7, 8**