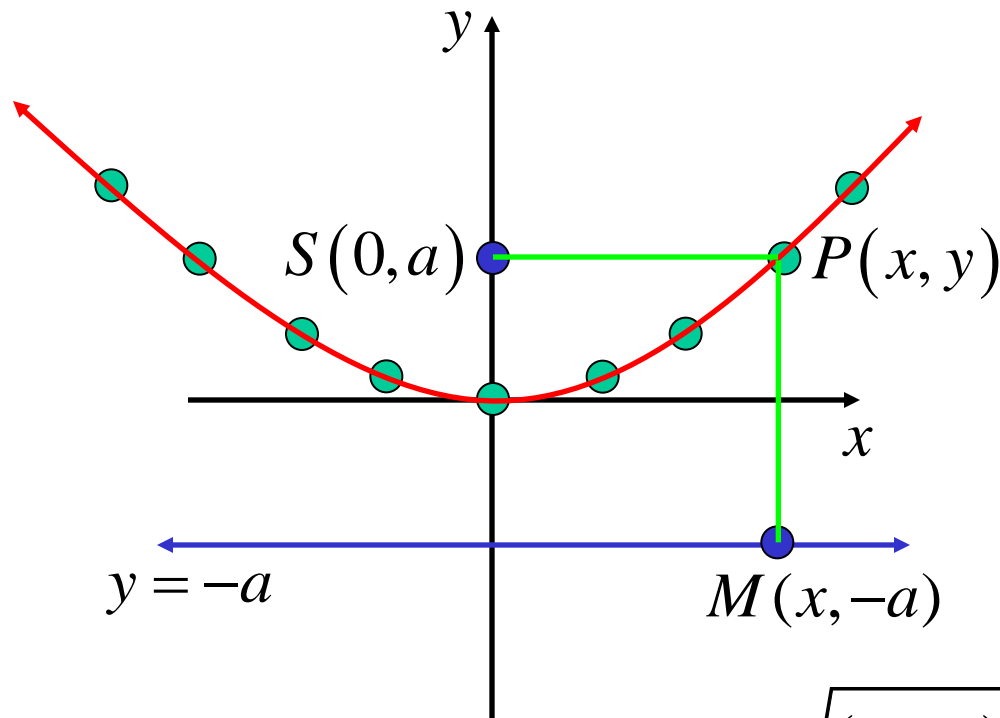


# *The Parabola As a Locus*

A point moves so that its distance from a fixed point (**focus**) is equal to its distance from a fixed line (**directrix**)



$$d_{PS} = d_{PM}$$

$$\sqrt{(x-0)^2 + (y-a)^2} = \sqrt{(x-x)^2 + (y+a)^2}$$

$$x^2 + (y-a)^2 = (y+a)^2$$

$$x^2 + y^2 - 2ay + a^2 = y^2 + 2ay + a^2$$

$$x^2 = 4ay$$

$$x^2 = 4ay$$

vertex: (0,0)

focus: (0, a)

directrix:  $y = -a$

focal length:  $a$  units

e.g. (i) Find the focus, focal length and directrix;

a)  $x^2 = 32y$

$$4a = 32$$

$$a = 8$$

focal length = 8 units

b)  $y = 4x^2 \Rightarrow x^2 = \frac{1}{4}y$

$$4a = \frac{1}{4}$$

$$a = \frac{1}{16}$$

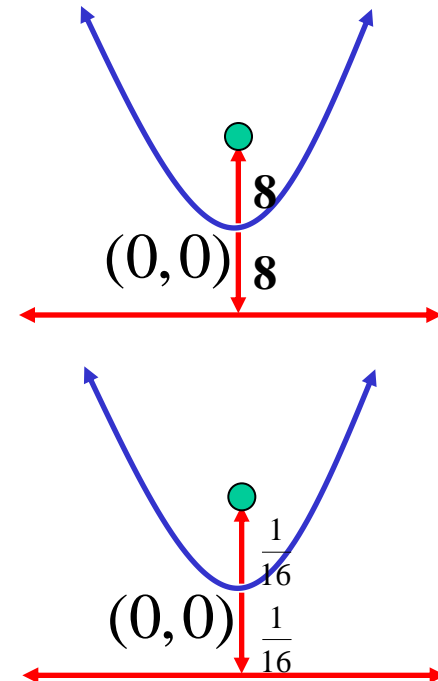
focus is (0,8)

directrix is  $y = -8$

focus is  $\left(0, \frac{1}{16}\right)$

directrix is  $y = -\frac{1}{16}$

focal length =  $\frac{1}{16}$  unit



(ii) Find the equation of the parabola with;

a) focus  $(0, -2)$ , directrix  $y = 2$

$$a = -2 \quad x^2 = 4(-2)y$$

$$\underline{x^2 = -8y}$$

b) focus  $(3, 0)$ , directrix  $x = -3$

$$a = 3 \quad y^2 = 4(3)x$$

$$\underline{y^2 = 12x}$$

**Vertex NOT at the origin**

$$(x - p)^2 = 4a(y - q)$$

vertex:  $(p, q)$

focus:  $(p, q + a)$

directrix:  $y = q - a$

focal length:  $a$  units

e.g. (i) Find the equation of the parabola with vertex (3,1) and focal length 2 units

$$(x-3)^2 = 4(2)(y-1)$$

$$(x-3)^2 = 8(y-1)$$

$$x^2 - 6x + 9 = 8y - 8$$

$$8y = x^2 - 6x + 17$$

$$\underline{y = \frac{1}{8}(x^2 - 6x + 17)}$$

$$(x-3)^2 = 4(-2)(y-1)$$

$$(x-3)^2 = -8(y-1)$$

$$x^2 - 6x + 9 = -8y + 8$$

$$8y = -x^2 + 6x - 1$$

$$\underline{y = -\frac{1}{8}(x^2 - 6x + 1)}$$

$$(y-1)^2 = 4(2)(x-3)$$

$$(y-1)^2 = 8(x-3)$$

$$y^2 - 2y + 1 = 8x - 24$$

$$8x = y^2 - 2y + 25$$

$$\underline{x = \frac{1}{8}(y^2 - 2y + 25)}$$

$$(y-1)^2 = 4(-2)(x-3)$$

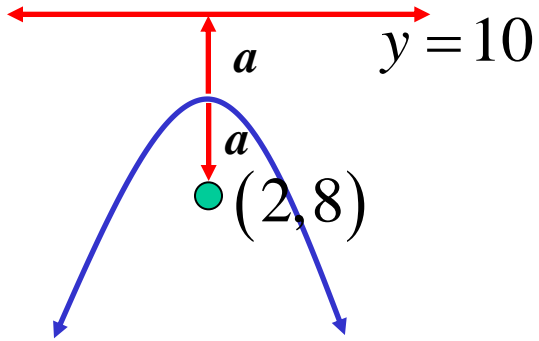
$$(y-1)^2 = -8(x-3)$$

$$y^2 - 2y + 1 = -8x + 24$$

$$8x = -y^2 + 2y + 23$$

$$\underline{x = -\frac{1}{8}(y^2 - 2y - 23)}$$

(ii) focus  $(2,8)$  and directrix  $y = 10$



$$2a = 2$$

$$a = 1 \quad \text{vertex is } (2,9)$$

$$(x-2)^2 = -4(1)(y-9)$$

$$(x-2)^2 = -4(y-9)$$

$$x^2 - 4x + 16 = -4y + 36$$

$$4y = -x^2 + 4x + 20$$

$$\underline{y = -\frac{1}{4}(x^2 - 4x - 20)}$$

(iii) Find the vertex, focus, focal length, directrix of  $12y = x^2 - 6x - 3$

$$12y = x^2 - 6x - 3$$

$$12y + 3 = x^2 - 6x$$

$$12y + 3 + 9 = (x - 3)^2$$

$$12y + 12 = (x - 3)^2$$

$$12(y + 1) = (x - 3)^2$$

$$4a = 12$$
$$a = 3$$

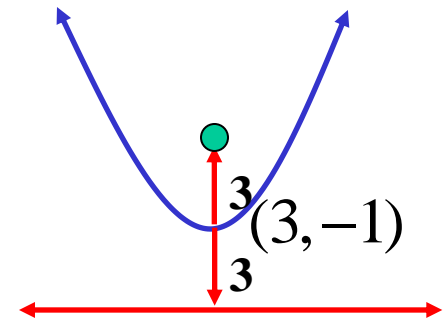
vertex:  $(3, -1)$

$\therefore$  focal length = 3 units

vertex =  $(3, -1)$

focus =  $(3, 2)$

directrix:  $y = -4$



**Exercise 9B; 1,2 try at home**

**4 (*use definition*)**

**6ace etc, 7ac, 8ace, 9ace, 10ac, 11bd, 12a**

**Exercise 9C; 3 to 8 ace etc, 10ac, 11ace, 12**