The Basic Counting Principle

If one event can happen in m different ways and after this another event can happen in n different ways, then the two events can occur in mn different ways.

e.g. 3 dice are rolled

(i) How many ways can the three dice fall?

the 1st die has 6 possibilities

the 2nd die has 6 possibilities

$\text{Ways} = 6 \times 6 \times 6$ ← the 3rd die has 6 possibilities

$= \underline{216}$

(ii) How many ways can all three dice show the same number?

the 1st die has 6 possibilities

the 2nd die now has only 1 possibility

$$\text{Ways} = 6 \times 1 \times 1$$

$$= 6$$

the 3rd die now has only 1 possibility

(iii) What is the probability that all three dice show the same number?

$$P(\text{all 3 the same}) = \frac{6}{216}$$

$$= \frac{1}{36}$$

1996 Extension 1 HSC Q5c)

Mice are placed in the centre of a maze which has five exits.

Each mouse is equally likely to leave the maze through any of the five exits. Thus, the probability of any given mouse leaving by a particular exit is $\frac{1}{5}$

Four mice, A, B, C and D are put into the maze and behave independently.

(i) What is the probability that A, B, C and D all come out the same exit?

First mouse can go through any door $\therefore P = 1$

Other mice must go through same door $\therefore P = \frac{1}{5}$

$$P(\text{all use the same exit}) = 1 \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$$

$$= \frac{1}{125}$$

(ii) What is the probability that A, B and C come out the same exit and D comes out a different exit?

D can go through any door $\therefore P = 1$

$$P(ABC \text{ use same exit}, D \text{ uses different exit}) = 1 \times \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5}$$

$$= \frac{4}{125}$$

Next mouse has 4 doors to choose

$\therefore P = \frac{4}{5}$

Other mice must go

through same door $\therefore P = \frac{1}{5}$

(iii) What is the probability that *any* of the four mice come out the same exit and the other comes out a different exit?

$$P(D \text{ uses different exit}) = \frac{4}{125}$$

$$\therefore P(A \text{ uses different exit}) = \frac{4}{125}$$

$$P(B \text{ uses different exit}) = \frac{4}{125}$$

$$P(C \text{ uses different exit}) = \frac{4}{125}$$

$$\therefore P(\text{any mouse uses different exit}) = 4 \times \frac{4}{125}$$

$$= \underline{\frac{16}{125}}$$

(iv) What is the probability that no more than two mice come out the same exit?

$$P(\text{no more than 2 use same exit}) = 1 - P(\text{all same}) - P(\text{3 use same})$$

$$= 1 - \frac{1}{125} - \frac{16}{125}$$

$$= \frac{108}{125}$$

Permutations

A permutation is an **ordered** set of objects

Case 1: Ordered Sets of *n* Different Objects, from a Set of *n* Such Objects

(i.e. use all of the objects)

If we arrange n different objects in a line, the number of ways we could arrange them are;

possibilities for object 1 — possibilities for object 2 — possibilities for object 3 — possibilities for last object

$$\text{Number of Arrangements} = n \times (n-1) \times (n-2) \times \cdots \times 1$$
$$= n!$$

e.g. In how many ways can 5 boys and 4 girls be arranged in a line if;

(i) there are no restrictions?

$$\text{Arrangements} = 9! = \underline{362880}$$

With no restrictions, arrange 9 people
gender does not matter

(ii) boys and girls alternate?

(ALWAYS look after any restrictions first)

first person MUST be a boy

number of ways of arranging the boys

$$\text{Arrangements} = 1 \times 5! \times 4! = \underline{2880}$$

number of ways of arranging the girls

(iii) What is the probability of the boys and girls alternating?

$$P(\text{boys \& girls alternate}) = \frac{2880}{362880}$$
$$= \frac{1}{126}$$

(iv) Two girls wish to be together?

the number of ways the girls can be arranged

number of ways of arranging 8 objects (2 girls) + 7 others

$$\text{Arrangements} = 2! \times 8!$$
$$= 80640$$

Case 2: Ordered Sets of *k* Different Objects, from a Set of *n* Such Objects ($k < n$)

(i.e. use some of the objects)

If we have *n* different objects in a line, but only want to arrange *k* of them, the number of ways we could arrange them are;

possibilities for object 1 — possibilities for object 2 — possibilities for object 3 — possibilities for object k

$$\text{Number of Arrangements} = n \times (n-1) \times (n-2) \times \cdots \times (n-k+1)$$

$$= n(n-1)(n-2)\cdots(n-k+1) \times \frac{(n-k)(n-k-1)\cdots(3)(2)(1)}{(n-k)(n-k-1)\cdots(3)(2)(1)}$$

$$= \frac{n!}{(n-k)!}$$

$$= {}^nP_k$$

e.g. (*i*) From the letters of the word **PROBLEMS** how many 5 letter words are possible if;

a) there are no restrictions?

$$\text{Words} = {}^{8}P_5$$
$$= 6720$$

b) they must begin with **P**?

the number of ways P can be placed first

$$\text{Words} = 1 \times {}^{7}P_4$$
$$= 840$$

Question now becomes how many 4 letter words ROBLEMS

c) **P** is included, but not at the beginning, and **M** is excluded?

the number of positions P can be placed in

Question now becomes how many 4 letter words ROBLES

$$\text{Words} = 4 \times {}^6P_4$$
$$= \underline{1440}$$

(*ii*) Six people are in a boat with eight seats, for on each side. What is the probability that Bill and Ted are on the left side and Greg is on the right?

$$\text{Ways (no restrictions)} = {}^8P_6$$
$$= 20160$$

Ways Bill & Ted can go

Ways Greg can go

Ways remaining people can go

$$\text{Ways (restrictions)} = {}^4P_2 \times {}^4P_1 \times {}^5P_3$$
$$= 2880$$

$$P(\text{B \& T left, G right}) = \frac{2880}{20160}$$
$$= \frac{1}{7}$$

2006 Extension 1 HSC Q3c)

Sophia has five coloured blocks: one red, one blue, one green, one yellow and one white.

She stacks two, three, four or five blocks on top of one another to form a vertical tower.

(i) How many different towers are there that she could form that are three blocks high?

$$\text{Towers} = {}^5P_3$$
$$= \underline{60}$$

(ii) How many different towers can she form in total?

$$2 \text{ block Towers} = {}^5P_2 \quad = 20$$
$$3 \text{ block Towers} = {}^5P_3 \quad = 60$$
$$4 \text{ block Towers} = {}^5P_4 \quad = 120$$
$$5 \text{ block Towers} = {}^5P_5 \quad = 120$$
$$\underline{\text{Total number of Towers} = 320}$$

Exercise 10E; odd *(not 39)*