Permutations

Case 3: Ordered Sets of *n* Objects, Not All Different

(i.e. some of the objects are the same)

2 objects

all different	*2 same*
A B	**A A**
B A	
$2! = 2$	1

3 objects

all different	*2 same*	*3 same*
A B C	**A A B**	**A A A**
A C B	**A B A**	
B A C	**B A A**	
B C A		
C A B		
C B A		
$3! = 6$	3	1

4 objects

all different

A	**B**	**C**	**D**	**C**	**B**	**A**	**D**
A	**B**	**D**	**C**	**C**	**B**	**D**	**A**
A	**C**	**B**	**D**	**C**	**A**	**B**	**D**
A	**C**	**D**	**B**	**C**	**A**	**D**	**B**
A	**D**	**B**	**C**	**C**	**D**	**B**	**A**
A	**D**	**C**	**B**	**C**	**D**	**A**	**B**
B	**A**	**C**	**D**	**D**	**A**	**C**	**B**
B	**A**	**D**	**C**	**D**	**A**	**B**	**C**
B	**C**	**A**	**D**	**D**	**C**	**A**	**B**
B	**C**	**D**	**A**	**D**	**C**	**B**	**A**
B	**D**	**A**	**C**	**D**	**B**	**A**	**C**
B	**D**	**A**	**B**	**D**	**B**	**A**	**B**

$4! = 24$

2 same

A	**A**	**B**	**C**
A	**A**	**C**	**B**
A	**B**	**A**	**C**
A	**B**	**C**	**A**
A	**C**	**A**	**B**
A	**C**	**B**	**A**
B	**A**	**A**	**C**
B	**A**	**C**	**A**
B	**C**	**A**	**A**
C	**A**	**A**	**B**
C	**A**	**B**	**A**
C	**B**	**A**	**A**

12

3 same

A	**A**	**A**	**B**
A	**A**	**B**	**A**
A	**B**	**A**	**A**
B	**A**	**A**	**A**

4

4 same

A	**A**	**A**	**A**

1

If we arrange n objects in a line, of which x are alike, the number of ways we could arrange them are;

$$\text{Number of Arrangements} = \frac{n!}{x!}$$

$n!$ — ways of arranging n objects

$x!$ — ways of arranging the like objects

e.g. How many different words can be formed using all of the letters in the word

CONNAUGHTON ?

$$\text{Words} = \frac{11!}{2!3!}$$

$$= 3326400$$

$2!$ for the two O's

$3!$ for the three N's

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The letters *A*, *E*, *I*, *O* and *U* are vowels

(*i*) How many arrangements of the letters in the word **ALGEBRAIC** are possible?

$$\text{Words} = \frac{9!}{2!}$$

$$= 181440$$

(*ii*) How many arrangements of the letters in the word **ALGEBRAIC** are possible if the vowels must occupy the 2nd, 3rd, 5th, and 8th positions?

$$\text{Words} = \frac{4!}{2!} \times 5!$$

$$= 1440$$

Number of ways of placing the vowels

Number of ways of placing the consonants

Exercise 10F; odd