Combinations

A combination is a set of objects where the order that they are arranged is not important.

If we arrange objects in a line, and the order is not important then;

A B is the same arrangement as **B A**

e.g. 5 objects, arrange 2 of them

A	**B**	~~**B**~~	~~**A**~~	**C**	~~**A**~~	**D**	~~**A**~~	**E**	~~**A**~~
A	**C**	**B**	**C**	~~**C**~~	~~**B**~~	**D**	~~**B**~~	**E**	~~**B**~~
A	**D**	**B**	**D**	**C**	**D**	~~**D**~~	~~**C**~~	**E**	~~**C**~~
A	**E**	**B**	**E**	**C**	**E**	**D**	**E**	~~**E**~~	~~**D**~~

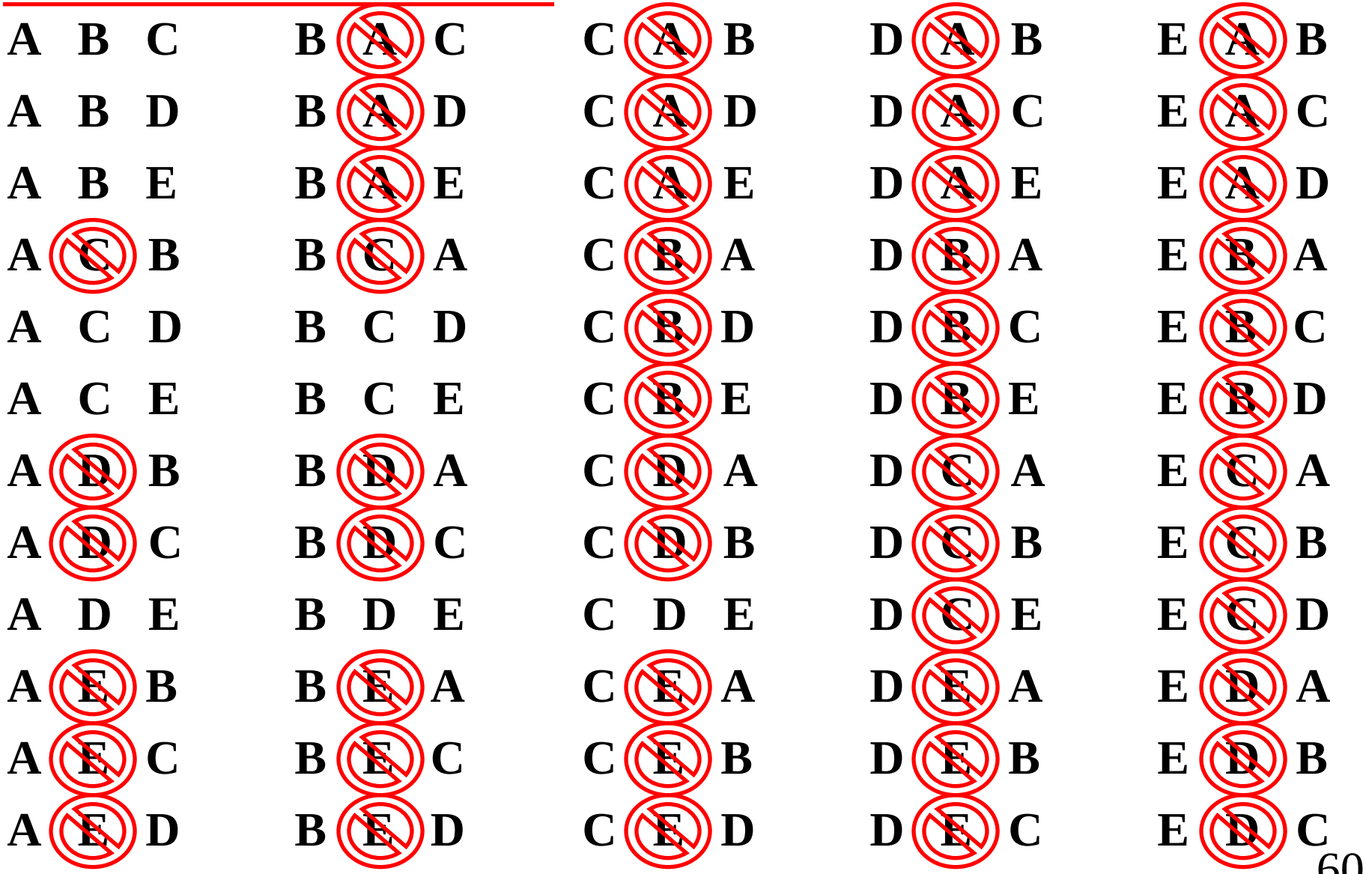
$\text{Permutations} = {}^{5}P_{2}$

$= 20$

$\text{Combinations} = \frac{20}{2!}$

$= 10$

5 objects, arrange 3 of them



Permutations $= {}^{5}P_{3}$

$= 60$

Combinations $= \frac{60}{3!}$

$= 10$

If we have n different objects, and we arrange k of them and are not concerned about the order;

$$\text{Number of Arrangements} = \frac{{}^{n}P_{k}}{k!}$$

$$= \frac{n!}{(n-k)!k!}$$

$$= {}^{n}C_{k}$$

e.g. (*i*) How many ways can 6 numbers be chosen from 45 numbers?

$$\text{Ways} = {}^{45}C_{6}$$

$$= 8145060$$

Note: at 40 cents per game, $3 258 024 = amount of money you have to spend to guarantee a win in Lotto.

(*ii*) Committees of five people are to be obtained from a group of seven men and four women.

How many committees are possible if;

a) there are no restrictions?

$\text{Committees} = {}^{11}C_5$

$= 462$

With no restrictions, choose 5 people from 11, gender does not matter

b) the committee contains only males?

$\text{Committees} = {}^{7}C_5$

$= 21$

By restricting it to only males, there is only 7 people to choose from

c) the committee contains at least one woman?

$\text{Committees} = 462 - 21$

$= 441$

easier to work out male only and subtract from total number of committees

(*iii*) A hand of five cards is dealt from a regular pack of fifty two cards.

a) What is the number of possible hands?

$$\text{Hands} = {}^{52}C_5$$
$$= 2598960$$

b) What is the probability of getting "three of a kind"?

choose which number has "three of a kind"

choose three of those cards

choose remaining two cards from the rest

$$\text{Hands} = {}^{13}C_1 \times {}^{4}C_3 \times {}^{48}C_2$$
$$= 58656$$

$$P(\text{three of a kind}) = \frac{58656}{2598960}$$
$$= \frac{94}{2915} \qquad (=3.2\%)$$

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A four person team is to be chosen at random from nine women and seven men.

(*i*) In how many ways can this team be chosen?

$$\text{Teams} = {}^{16}C_4$$
$$= 1820$$

With no restrictions, choose 4 people from 16, gender does not matter

(*ii*) What is the probability that the team will consist of four women?

$$\text{Teams} = {}^{9}C_4$$
$$= 126$$

By restricting it to only women, there is only 9 people to choose from

$$P(\text{4 women team}) = \frac{126}{1820}$$
$$= \frac{9}{130}$$

Exercise 10G; odd
(not 19, 27)