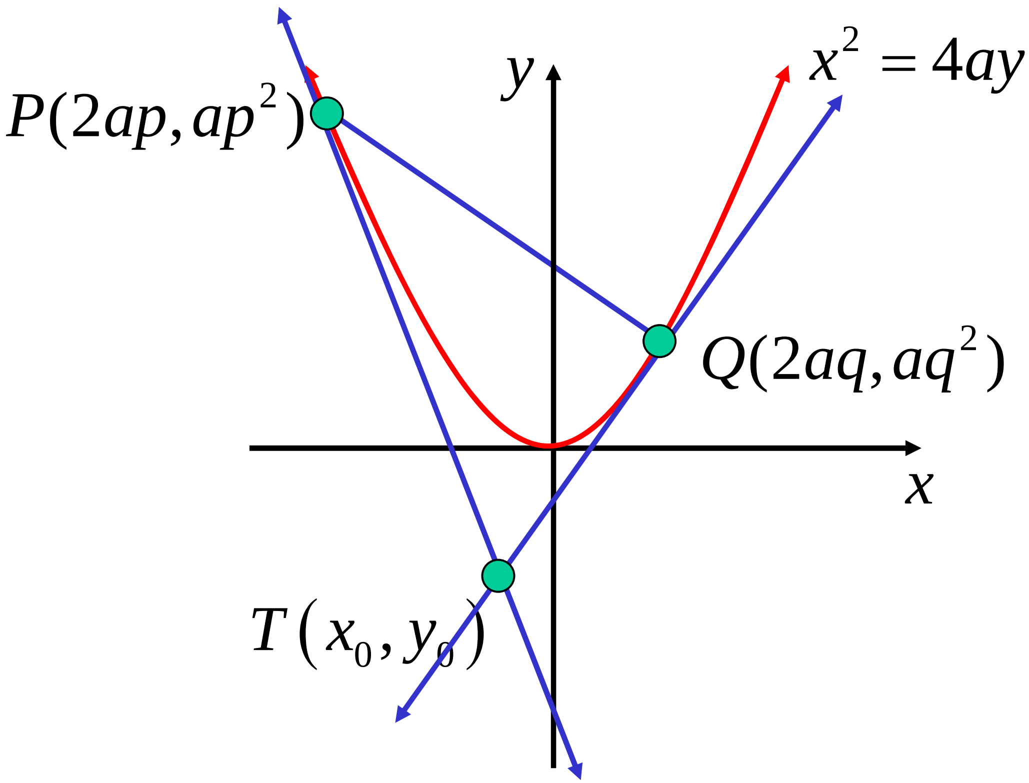
Chord of Contact



We know the coordinates of an external point (*T*)

From this external point, two tangents can be drawn meeting the parabola at *P* and *Q*.

The line joining these two points is called the **chord of contact.**

(1) Parametric approach

1 Show that PQ has equation $(p+q)x-2y=2apq$

2 Show the two tangents have equations

$px-y-ap^2=0$ and $qx-y-aq^2=0$

3 Show that T is the point $\{a(p+q), apq\}$

4 But T is (x_0, y_0)

$$\therefore x_0=a(p+q) \qquad \therefore y_0=apq$$

$$p+q=\frac{x_0}{a}$$

$$PQ \text{ is } \frac{x_0 x}{a}-2y=2y_0$$

notice similarity to tangent

Hence the chord of contact is $x_0 x=2a(y_0+y)$

(2) Cartesian approach

1 Show that PT has equation $xx_1 = 2a(y + y_1)$

T lies on PT $\therefore x_0 x_1 = 2a(y_0 + y_1)$

$\therefore P(x_1, y_1)$ lies on the line with equation $x_0 x = 2a(y_0 + y)$

2 Show that QT has equation $xx_2 = 2a(y + y_2)$

T lies on QT $\therefore x_0 x_2 = 2a(y_0 + y_2)$

$\therefore Q(x_2, y_2)$ lies on the line with equation $x_0 x = 2a(y_0 + y)$

Hence the chord of contact is $x_0 x = 2a(y_0 + y)$

Exercise 9H; 1c, 2d, 3, 6, 8, 10, 14