Sum and Product of Roots

If α and β are the roots of $ax^2+bx+c=0,$ then;

$$ax^2+bx+c=a(x-\alpha)(x-\beta)$$

$$ax^2+bx+c=a\left(x^2-\alpha x-\beta x+\alpha\beta\right)$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=x^2-(\alpha+\beta)x+\alpha\beta$$

Thus

$$\alpha+\beta=\frac{-b}{a} \qquad \text{(sum of roots)}$$

$$\alpha\beta=\frac{c}{a} \qquad \text{(product of roots)}$$

e.g. (i) Form a quadratic equation whose roots are;

a) 2 and -3

$\alpha + \beta = -1$

$\alpha\beta = -6$

$x^2 + x - 6 = 0$

b) $2+\sqrt{5}$ and $2-\sqrt{5}$

$\alpha + \beta = 4$

$\alpha\beta = 4 - 5$

$= -1$

$x^2 - 4x - 1 = 0$

(*ii*) If α and β are the roots of $2x^2 - 3x - 1 = 0$, find;

a) $\alpha + \beta = \dfrac{-b}{a}$

$= \dfrac{3}{2}$

b) $\alpha\beta = \dfrac{c}{a}$

$= \dfrac{-1}{2}$

$c)\ \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(\frac{3}{2}\right)^2-2\left(\frac{-1}{2}\right)$

$=\frac{9}{4}+1$

$=\frac{13}{4}$

$d)\ \frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{\frac{3}{2}}{\frac{-1}{2}}$

$=-3$

(*iii*) Find the value of m if one root is double the other in $x^2+6x+m=0$

Let the roots be α and 2α

$\alpha+2\alpha=-6$

$3\alpha=-6$

$\alpha=-2$

$(\alpha)(2\alpha)=m$

$2\alpha^2=m$

$2(-2)^2=m$

$m=8$

(*iv*) Find the values of m in $(2m-1)x^2+(1+m)x+1=0$, if one root is the reciprocal of the other.

Let the roots be α and $\frac{1}{\alpha}$

$$(\alpha)\left(\frac{1}{\alpha}\right)=\frac{1}{2m-1}$$

$$1=\frac{1}{2m-1}$$

$$2m-1=1$$

$$2m=2$$

$$m=1$$

Exercise 8H; 3beh, 4bdf, 5, 6, 7abc *i*, 8, 10, 12, 13cd, 15, 18, 20, 21ac