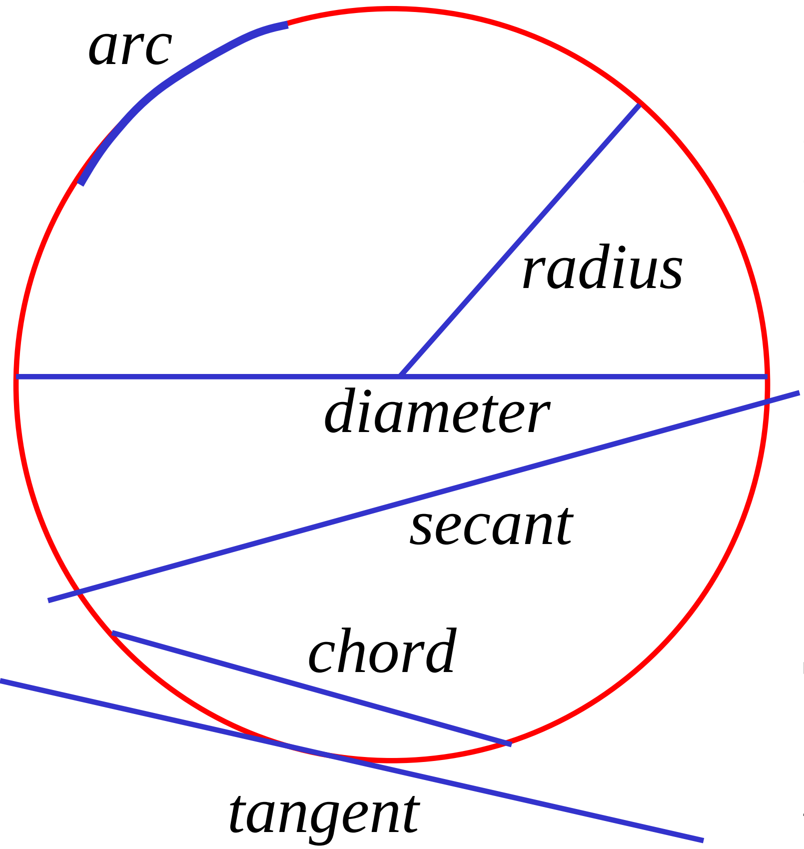
Circle Geometry

Circle Geometry Definitions



Radius: an interval joining centre to the circumference

Diameter: an interval passing through the centre, joining any two points on the circumference

Chord: an interval joining two points on the circumference

Secant: a line that cuts the circle

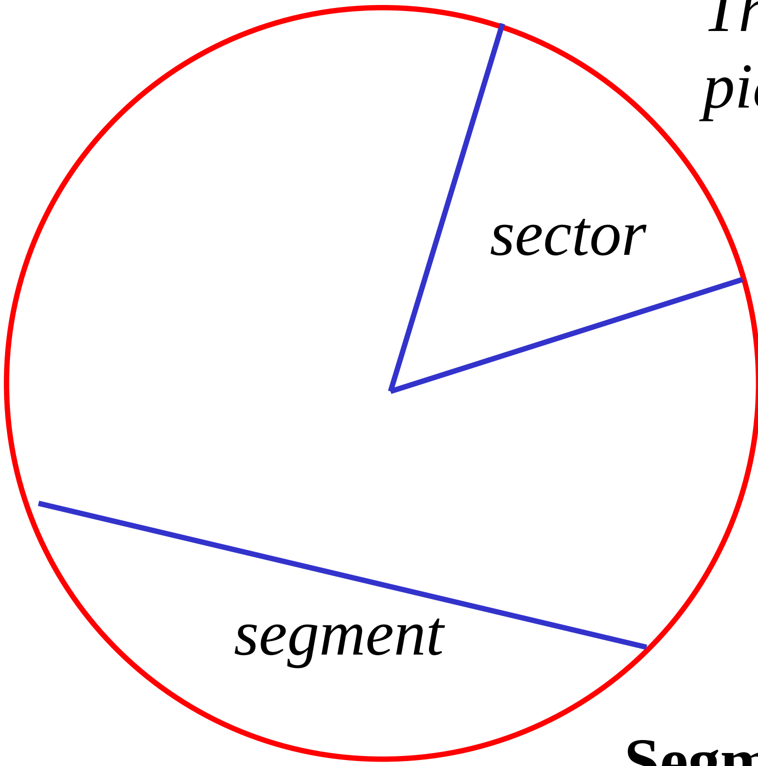
Tangent: a line that touches the circle

Arc: a piece of the circumference

Sector: a plane figure with two radii and an arc as boundaries.

The minor sector is the small "piece of the pie", the major sector is the large "piece"

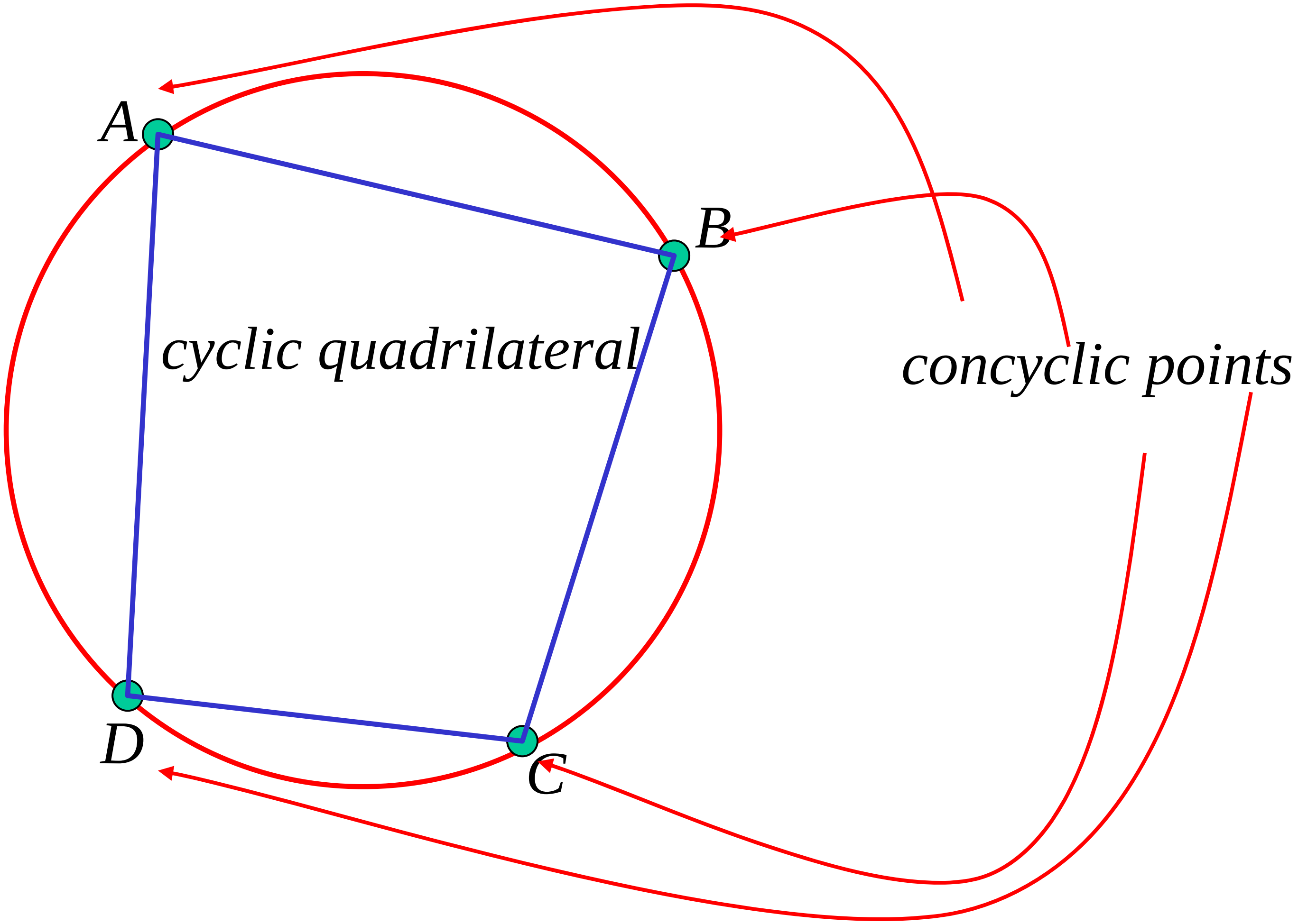
A quadrant is a sector where the angle at the centre is 90 degrees



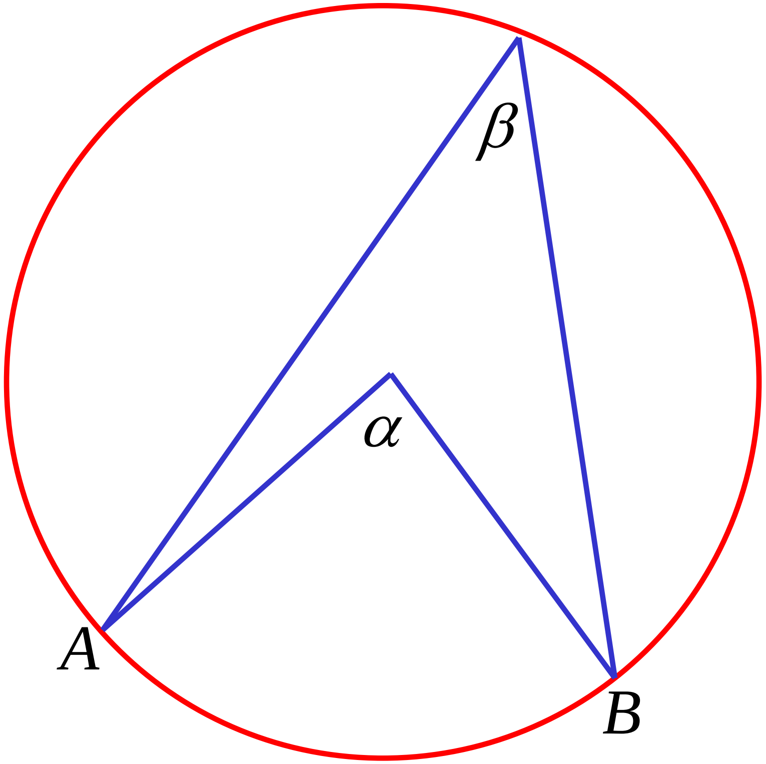
Segment: a plane figure with a chord and an arc as boundaries.

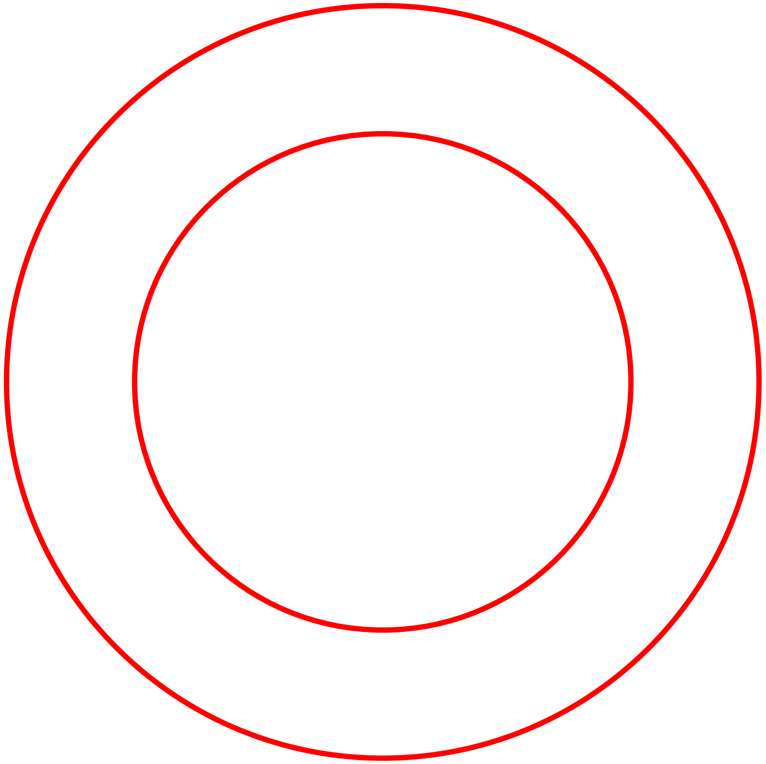
A semicircle is a segment where the chord is the diameter, it is also a sector as the diameter is two radii.

Concyclic Points: points that lie on the same circle.



Cyclic Quadrilateral: a four sided shape with all vertices on the same circle.

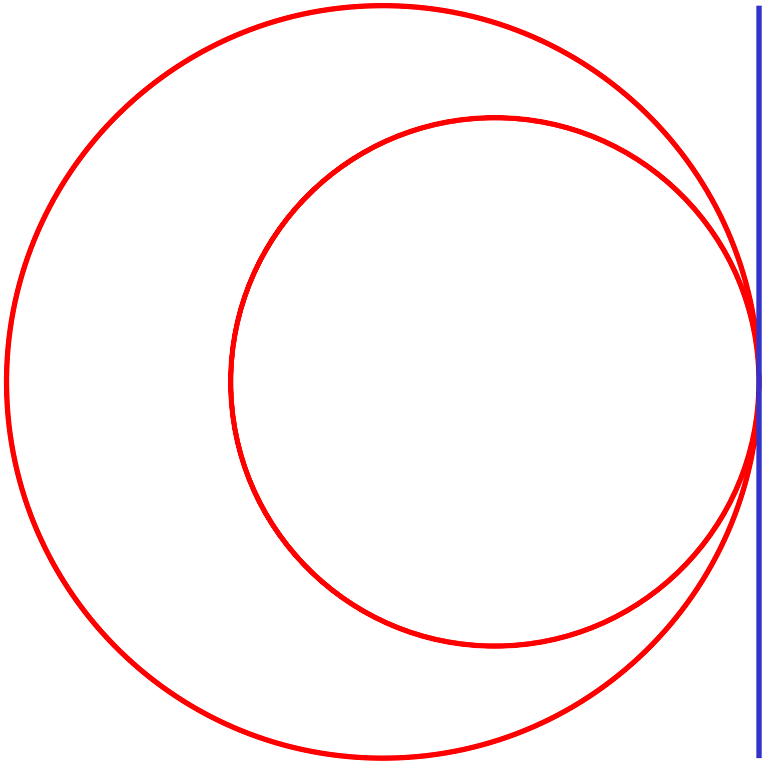




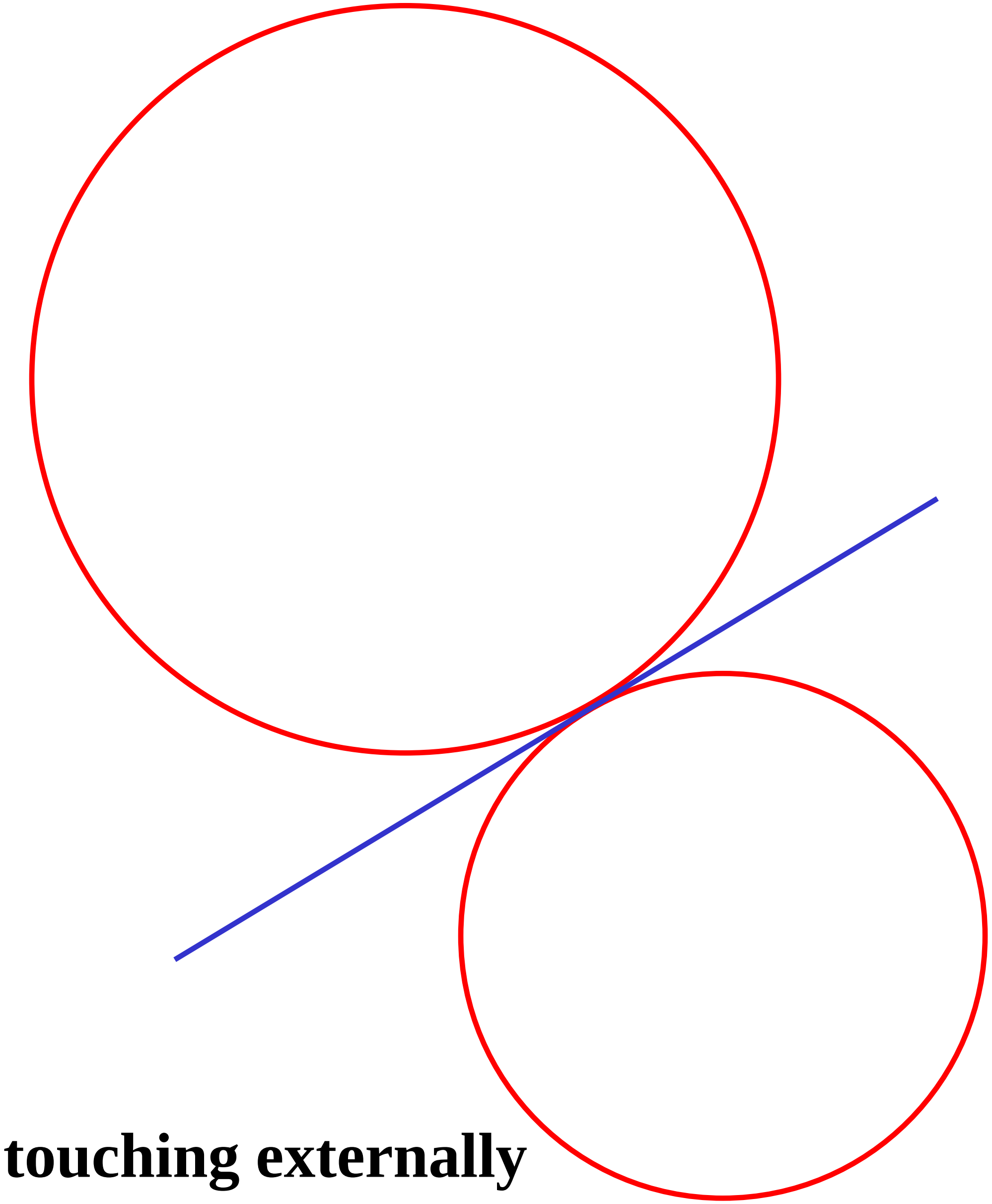
α represents the **angle subtended** at the centre by the arc AB

β represents the **angle subtended** at the circumference by the arc AB

Concentric circles have the same centre.



Circles touching internally
share a common tangent.



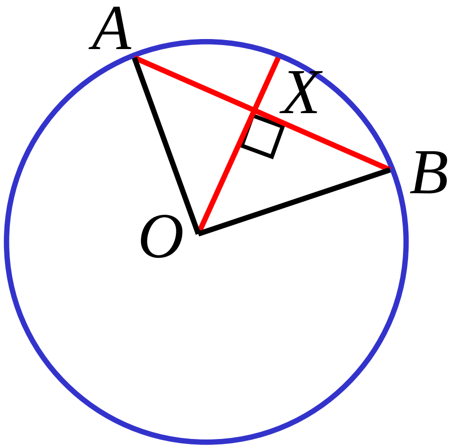
Circles touching externally
share a common tangent.

Chord (Arc) Theorems

Note: *= chords cut off = arcs*

(1) A perpendicular drawn to a chord from the centre of a circle bisects the chord, and the perpendicular bisector of a chord passes through the centre.

$AX = BX$ $\quad$ $(\perp \text{ from centre, bisects chord})$



Data : $AB \perp OX$

Prove : $AX = BX$

Proof: Join OA, OB

$\angle AXO = \angle BXO = 90^\circ$ $\quad$ (given)(R)

$AO = BO$ $\quad$ $(= \text{radii})(H)$

OX is common $\quad$ (S)

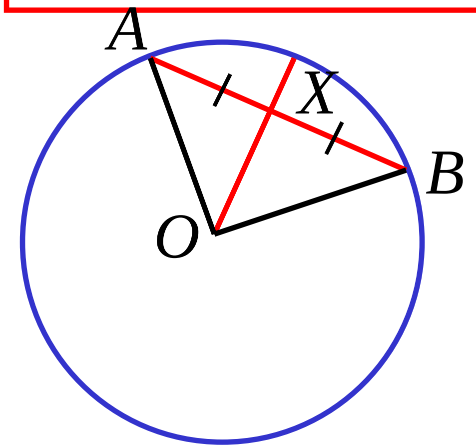
$\therefore \Delta AXO \equiv \Delta BXO$ $\quad$ (RHS)

$\therefore AX = BX$ $\quad$ (matching sides in $\equiv \Delta$'s)

(2) *Converse of (1)*

The line from the centre of a circle to the midpoint of the chord at right angles.

$OX \perp AB$ (line joining centre to midpoint, $\perp$ to chord)



Data: $AX = BX$

Prove: $AB \perp OX$

Proof: Join OA, OB

$AX = BX$ (given)(S)

$AO = BO$ (= radii)(S)

OX is common (S)

$\therefore \Delta AXO \equiv \Delta BXO$ (SSS)

$\therefore \angle AXO = \angle BXO$ (matching $\angle$'s in $\equiv \Delta$'s)

$\angle AXO + \angle BXO = 180^\circ$ (straight $\angle$ AXB)

$2\angle AXO = 180^\circ$

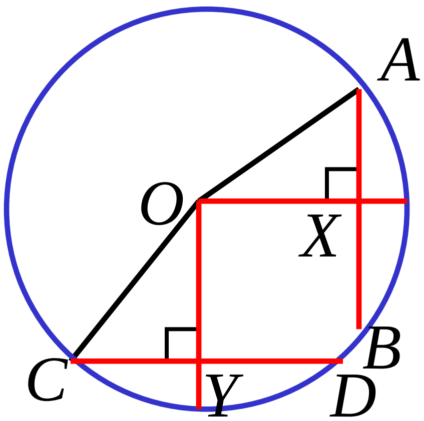
$\angle AXO = 90^\circ$

$\therefore AB \perp OX$

(3) Equal chords of a circle are the same distance from the centre and subtend equal angles at the centre.

$$OX = OY \qquad (= \text{chords, equidistant from centre})$$

$$\angle AOB = \angle COD \qquad (= \text{chords subtend} = \angle\text{'s at centre})$$



Data: $AB = CD, OX \perp AB, OY \perp CD$

Prove: $OX = OY$

Proof: Join OA, OC

$$AB = CD \qquad (\text{given})$$

$$AX = \frac{1}{2}AB \qquad (\perp \text{ bisects chord})$$

$$CY = \frac{1}{2}CD \qquad (\perp \text{ bisects chord})$$

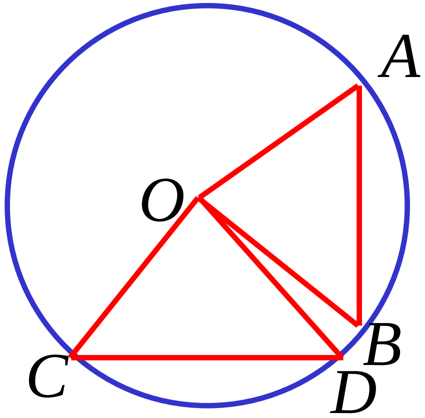
$$\therefore AX = CY \qquad (S)$$

$$\angle AXO = \angle CYO = 90^\circ \qquad (\text{given})(R)$$

$$OA = OC \qquad (= \text{radii})(H)$$

$$\therefore \Delta AXO \equiv \Delta CYO \qquad (RHS)$$

$$\therefore OX = OY \qquad (\text{matching sides in } \equiv \Delta\text{'s})$$



Data : $AB = CD$

Prove : $\angle AOB = \angle COD$

Proof: $AB = CD$ (given)

$AO = BO = CO = DO$ (= radii)

$\therefore \Delta AOB \equiv \Delta COD$ (SSS)

$\therefore \angle AOB = \angle COD$ (matching $\angle$'s in $\equiv \Delta$'s)

Exercise 9A; 1 ce, 2 aceg, 3, 4, 9, 10 ac, 11 ac, 13, 15, 16, 18