Geometrical Applications of Differentiation

The First Derivative $y', f'(x), \frac{d}{dx}\{f(x)\}, \frac{dy}{dx}$

$\frac{dy}{dx}$ measures the slope of the tangent to a curve

If $f'(x) > 0$, the curve is increasing
If $f'(x) < 0$, the curve is decreasing
If $f'(x) = 0$, the curve is stationary

e.g. For the curve $y = 3x^2 - x^3$, find all of the stationary points and determine their nature.

Hence sketch the curve

$$\frac{dy}{dx} = 6x - 3x^2$$

Stationary points occur when $\frac{dy}{dx} = 0$

$i.e.\ \ 6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0 \quad \text{or} \quad x = 2$

$\therefore$ stationary points occur at $(0,0)$ and $(2,4)$

$(0,0)$

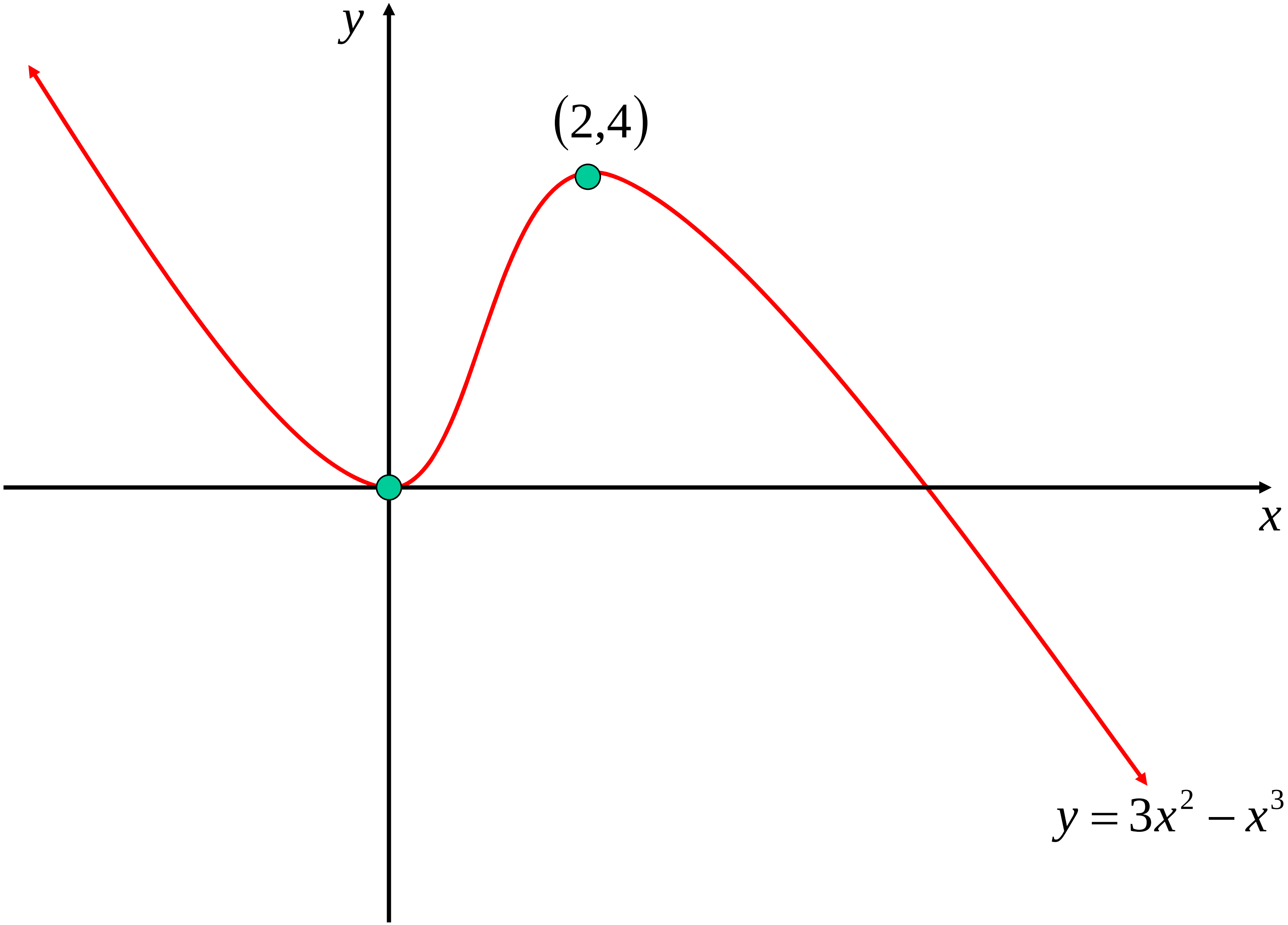
x	$0^-_{(-1)}$	0	$0^+_{(1)}$
$\frac{dy}{dx}$	(-9) \	0 —	(3) /

$\therefore (0,0)$ is a minimum turning point

$(2,4)$

x	$2^-_{(1)}$	2	$2^+_{(3)}$
$\frac{dy}{dx}$	(3) /	0 —	(-9) \

$\therefore (2,4)$ is a maximum turning point


y
(2,4)
x
$y = 3x^2 - x^3$

Exercise 10A; 1, 2ace, 4, 5, 6ac, 7, 8, 9ace etc, 11, 12, 13, 15ace, 16, 17

Exercise 10B; 1ad, 2ac, 3ac, 5, 6, 7ac, 8, 10, 12