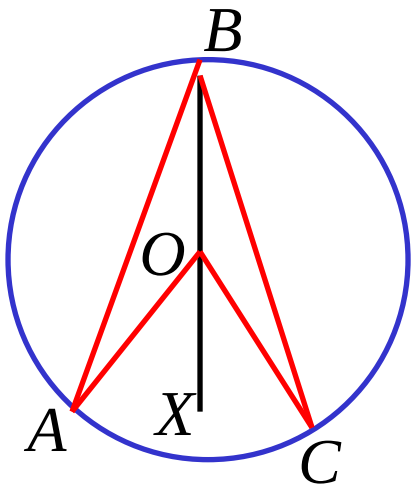
Angle Theorems

(4) The angle subtended by an arc (or chord) at the centre is double the angle subtended by the arc (or chord) at the circumference.

$\angle AOC = 2\angle ABC$ ($\angle$at centre, twice $\angle$at circumference on same arc)



Prove : $\angle AOC = 2\angle ABC$

Proof: Join BO and produce to X

ΔAOB is isosceles ($OA = OB$, = radii)

$\therefore \angle OBA = \angle OAB$ (base$\angle$'s isoscelesΔ)

$\angle AOX = \angle OBA + \angle OAB$ (exterior$\angle$ ΔOAB)

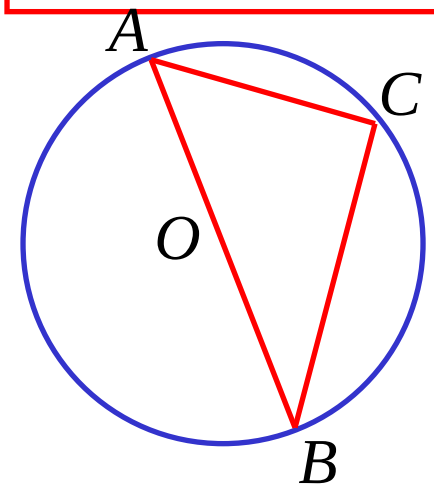
$\therefore \angle AOX = 2\angle OBA$

$\therefore \angle COX = 2\angle OBC$ (by similar method)

$\therefore \angle AOC = 2\angle ABC$

(5a) The angle in a semicircle is a right angle.

$$\angle ACB = 90^\circ \qquad (\angle \text{in a semicircle})$$



Data : AOB is diameter

Prove : $\angle ACB = 90^\circ$

Proof: $\angle AOB = 180^\circ$ $(\text{straight}\angle)$

$\angle AOB = 2\angle ACB$ $(\angle \text{at centre twice} \angle \text{at circumference on same arc})$

$\angle ACB = 90^\circ$

Exercise 9B; 1 ace etc, 2, 6, 8ac, 9ab, 10ac, 11ac, 12, 13