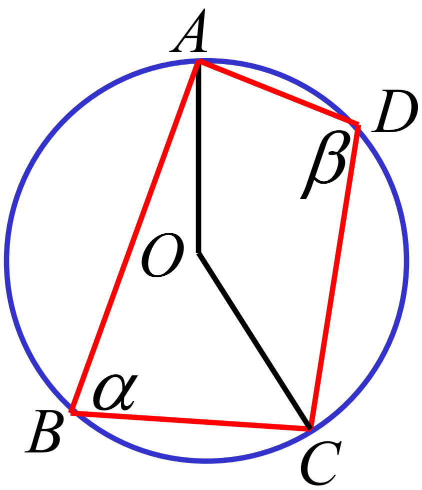
Angle Theorems

(5b) Opposite angles of a cyclic quadrilateral are supplementary.

$\angle ABC + \angle ADC = 180^\circ$ $\quad$ (opposite $\angle$'s in cyclic quadrilateral $= 180^\circ$)



Prove: $\alpha + \beta = 180^\circ$

Proof: Join AO and CO

$\angle AOC = 2\alpha$ $\quad$ ($\angle$at centre twice $\angle$at circumferenceon same arc)

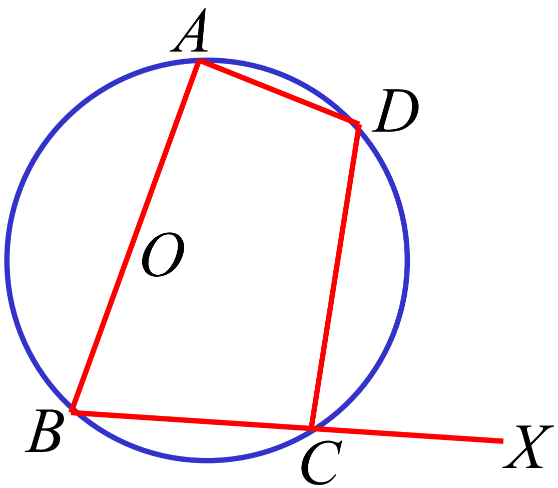
$\angle AOC_{reflex} = 2\beta$ $\quad$ ($\angle$at centre twice $\angle$at circumferenceon same arc)

$\therefore \angle AOC + \angle AOC_{reflex} = 2\alpha + 2\beta = 360^\circ$ $\quad$ (revolution)

$\therefore \alpha + \beta = 180^\circ$

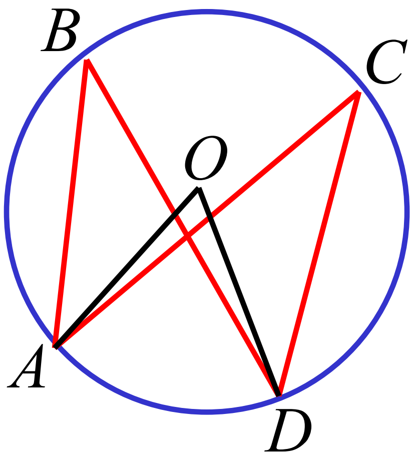
(5c) The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

$$\angle DCX = \angle BAD \qquad \begin{pmatrix} \text{exterior}\angle\text{cyclic quadilateral} \\ = \text{opposite interior}\angle \end{pmatrix}$$



(6) Angles subtended at the circumference by the same or equal arcs (or chords) are equal.

$\angle ABD = \angle ACD$ $\quad$ ($\angle$'s in same segment are =)



Prove: $\angle ABD = \angle ACD$

Proof: Join AO and DO

$\angle AOD = 2\angle ABD$ $\quad$ ($\angle$ at centre twice $\angle$ at circumference on same arc)

$\angle AOD = 2\angle ACD$ $\quad$ ($\angle$ at centre twice $\angle$ at circumference on same arc)

$\therefore \angle ABD = \angle ACD$

Exercise 9C; 1 ace etc, 2 aceg, 3, 4 bd, 6 bdf, 7 bdf, 9, 13, 14