

# Curve Sketching

## Look for;

- points of discontinuity e.g.  $y = \frac{1}{x}, x \neq 0, y \neq 0$
- asymptotes e.g.  $y = x + \frac{1}{x}, x \neq 0, y \neq x$

## On the curve $y = f(x)$

- (1) stationary points occur when  $f'(x) = 0$
- (2) maximum turning point if  $f'(x) = 0, f''(x) < 0$
- (3) minimum turning point if  $f'(x) = 0, f''(x) > 0$
- (4) point of inflection if  $f''(x) = 0$  and there is a change in concavity [ $f'''(x) \neq 0$ ]

- A curve is;
- increasing if  $f'(x) > 0$
  - decreasing if  $f'(x) < 0$
  - concave up if  $f''(x) > 0$
  - concave down if  $f''(x) < 0$

e.g. Sketch the curve  $y = x^3 - 6x^2 + 9x - 5$

$$\frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\frac{d^2y}{dx^2} = 6x - 12$$

$$\frac{d^3y}{dx^3} = 6$$

Stationary points occur when  $\frac{dy}{dx} = 0$

$$\text{i.e. } 3x^2 - 12x + 9 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1 \quad \text{or} \quad x = 3$$

$$\text{when } x = 1, \frac{d^2y}{dx^2} = 6(1) - 12$$

$$= -6 < 0$$

$\therefore (1, -1)$  is a maximum turning point

$$\text{when } x = 3, \frac{d^2 y}{dx^2} = 6(3) - 12 \\ = 6 > 0$$

$\therefore (3, -5)$  is a minimum turning point

Possible points of inflection occur when  $\frac{d^2 y}{dx^2} = 0$

*i.e.*  $6x - 12 = 0$

$$x = 2$$

$$\text{when } x = 2, \frac{d^3 y}{dx^3} = 6 \neq 0$$

$\therefore (2, -3)$  is a point of inflection

**Exercise 10F;**  
**1bdf, 2, 4b,**  
**5ace etc, 6bd**

**Exercise 10G**  
**1, 2ace, 4b, 5c**

