

# *Sum Of An Arithmetic Series*

$$S_n = a + a + d + a + 2d + \dots + l - 2d + l - d + l$$

$$S_n = \underline{l + l - d + l - 2d + \dots + a + 2d + a + d + a}$$

$$2S_n = a + l + a + l + a + l + \dots + a + l + a + l + a + l \\ = n(a + l)$$

$$S_n = \frac{n}{2}(a + l)$$

if we know  $l$

otherwise;

$$S_n = \frac{n}{2}\{a + a + (n - 1)d\}$$

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

e.g. (i) If  $a = 3$  and  $T_6 = 96$ , find  $S_6$

$$S_n = \frac{n}{2}(a + l)$$

$$\begin{aligned} S_6 &= \frac{6}{2}(3 + 96) \\ &= \underline{297} \end{aligned}$$

(ii) Find the sum of the first 100 even numbers

$$a = 2, d = 2 \text{ and } n = 100$$

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

$$\begin{aligned} S_{100} &= \frac{100}{2}\{4 + (99)(2)\} \\ &= 50 \times 202 \\ &= \underline{10100} \end{aligned}$$

(iii) The sum of the first 10 numbers is 100 and the first 5 numbers is 25.

Find  $a, d$  and the general term.

$$\begin{array}{rcl} S_{10} = 100 & & S_5 = 25 \\ \frac{10}{2} \{2a + 9d\} = 100 & & \frac{5}{2} \{2a + 4d\} = 25 \\ 2a + 9d = 20 & & a + 2d = 5 \end{array}$$

$$2a + 9d = 20$$

$$\underline{2a + 4d = 10}$$

$$5d = 10$$

$$d = 2 \quad \therefore a = 1$$

$$T_n = a + (n-1)d$$

$$= 1 + (n-1)2$$

$$= 2n - 1$$

$$\underline{\therefore a = 1, d = 2, T_n = 2n - 1}$$

$$(iv) \sum_{n=1}^{10} (3n - 6)$$

$$a = -3, l = 24, n = 10$$

$$S_n = \frac{n}{2}(a + l)$$

$$\begin{aligned} S_{10} &= \frac{10}{2}(-3 + 24) \\ &= \underline{105} \end{aligned}$$

**Exercise 6I; 2ace, 4bd, 5ac, 7c, 8f, 9, 11ac, 13be, 16a  
17bc, 19, 20a\***