Sum Of A Geometric Series

$$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

$$rS_n = \qquad ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n$$

$$(r-1)S_n = ar^n - a$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{, if } |r| > 1$$

$$OR$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad \text{, if } |r| < 1$$

Sum To Infinity (Limiting Sum)

$NOTE: |r| < 1$

If $|r| < 1$, $\quad \lim_{n \to \infty} r^n = 0$

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} \frac{a(1 - r^n)}{1 - r}$$

$$= \frac{a}{1 - r}$$

$$S_\infty = \frac{a}{1 - r}, \text{ if } |r| < 1$$

$e.g. (i)$ Find the sum of the first 10 terms of $2 + 6 + 18 + \dots$

$a = 2, r = 3$ and $n = 10$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{2(3^{10} - 1)}{3 - 1}$$

$$= 59048$$

$(ii)\ \sum_{n=3}^{8} 6\left(\frac{1}{2}\right)^{n-1}$

$$a = 6\left(\frac{1}{2}\right)^{2} \qquad r = \frac{1}{2},\ n = 6$$

$$= \frac{3}{2}$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S_6 = \frac{\frac{3}{2}\left(1-\left(\frac{1}{2}\right)^{6}\right)}{1-\frac{1}{2}}$$

$$= \frac{3}{2}\times\frac{63}{64}\times\frac{2}{1}$$

$$= \frac{189}{64}$$

(iii) Does $56+4+\frac{2}{7}+\ldots$ have a limiting sum?

$$r = \frac{4}{56} < 1$$

$\therefore$ as $|r|<1$, it has a limiting sum

$$(iv)\ \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$$

$$a=\frac{1}{2}, r=\frac{1}{2}$$

$$S_\infty=\frac{a}{1-r}$$

$$=\frac{\frac{1}{2}}{1-\frac{1}{2}}$$

$$=1$$

(v) Write $0.\dot{3}\dot{6}$ as a fraction

$$0.\dot{3}\dot{6}=0.36+0.0036+0.000036+\ldots$$

$$a=0.36, r=0.01$$

$$S_\infty=\frac{a}{1-r}$$

$$=\frac{0.36}{1-0.01}$$

$$=\frac{36}{99}$$

$$=\frac{4}{11}$$

Exercise 6J; 3cf, 5, 7b 8a(i), 9, 12, 14, 17b

Exercise 6K; 2adgj, 3b, 4a, 6bf, 8, 11a, 17ace, 19*

Exercise 6L, 1dg, 2d, 4a