Locus Problems

Eliminate the parameter (get rid of the p's and q's)

We find the locus of a point.

1 Find the coordinates of the point whose locus you are finding.

2 Look for the relationship between the x and y values
(*i.e. get rid of the parameters)*

a) <u>Type 1</u>: already no parameter in the x or y value.

e.g. $(p + q, -3)$

locus is $y = -3$

b) <u>Type 2</u>: obvious relationship between the values, usually only one parameter.

e.g. $\left(6t, t^2 + 1\right)$

$x = 6t$ $\qquad$ $y = t^2 + 1$

$t = \frac{x}{6}$ $\qquad$ $y = \frac{x^2}{36} + 1$

c) Type 3: not an obvious relationship between the values, use a previously proven relationship between the parameters.

e.g. $\left(p^2+q^2, p+q\right)$ $\quad x = p^2+q^2$

Given that $pq = 3$ $\quad = (p+q)^2 - 2pq$

$x = y^2 - 6$

2005 Extension 1 HSC Q4c)

The points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ lie on the parabola $x^2 = 4ay$

The equation of the normal to the parabola at P is $x + py = 2ap + ap^3$

and the equation of the normal at Q is given by $x + qy = 2aq + aq^3$

(i) Show that the normals at P and Q intersect at the point R whose coordinates are $\left\{-apq(p+q), a\left(p^2+pq+q^2+2\right)\right\}$

(ii) The equation of the chord PQ is $y = \frac{1}{2}(p+q)x - apq$. If PQ passes through $(0,a)$, show that $pq = -1$

(iii) Find the locus of R if PQ passes through $(0,a)$

$$x=-apq(p+q)$$

$$x=a(p+q)$$

$$p+q=\frac{x}{a}$$

$$y=a\left(p^2+pq+q^2+2\right)$$

$$y=a\left((p+q)^2-pq+2\right)$$

$$y=a\left(\frac{x^2}{a^2}+1+2\right)$$

$$y=\frac{x^2}{a}+3a$$

2004 Extension 1 HSC Q4b)

The two points $P\left(2ap, ap^2\right)$ and $Q\left(2aq, aq^2\right)$ are on the parabola $x^2 = 4ay$

(i) The equation of the tangent to $x^2 = 4ay$ at an arbitrary point $\left(2at, at^2\right)$ on the parabola is $y = tx - at^2$

Show that the tangents at the points P and Q meet at R, where R is the point $\{a(p+q), apq\}$

(ii) As P varies, the point Q is chosen so that $\angle POQ$ is a right angle, where O is the origin.

Find the locus of R.

$$m_{OP} \times m_{OQ} = -1$$

$$\frac{ap^2 - 0}{2ap - 0} \times \frac{aq^2 - 0}{2aq - 0} = -1$$

$$ap^2q^2 = -4a^2pq$$

$$pq = -4$$

$$y = apq$$

$$y = -4a$$

Exercise 9J; odds up to 23