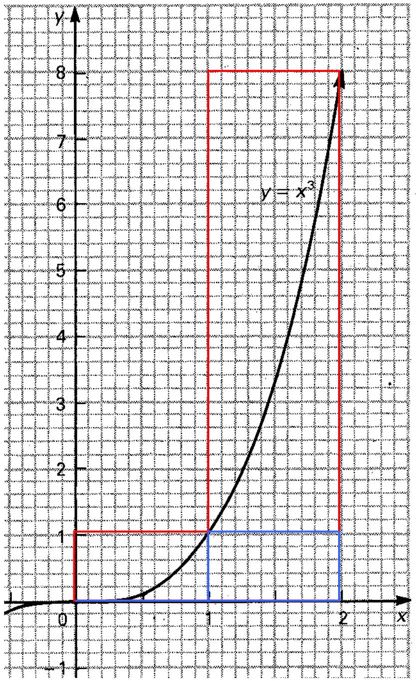
Integration
Area Under Curve

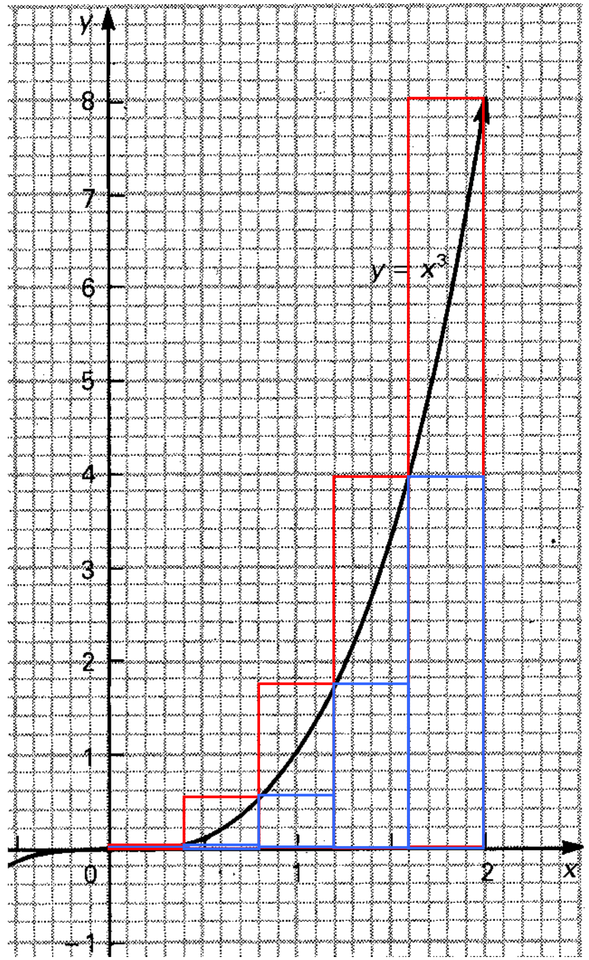


$$1(0)^3 + 1(1)^3 \leq \text{ Area} \leq 1(1)^3 + 1(2)^3$$

$$1 \leq \text{ Area} \leq 9$$

$$\text{Estimate Area} = 5\,\text{unit}^2$$

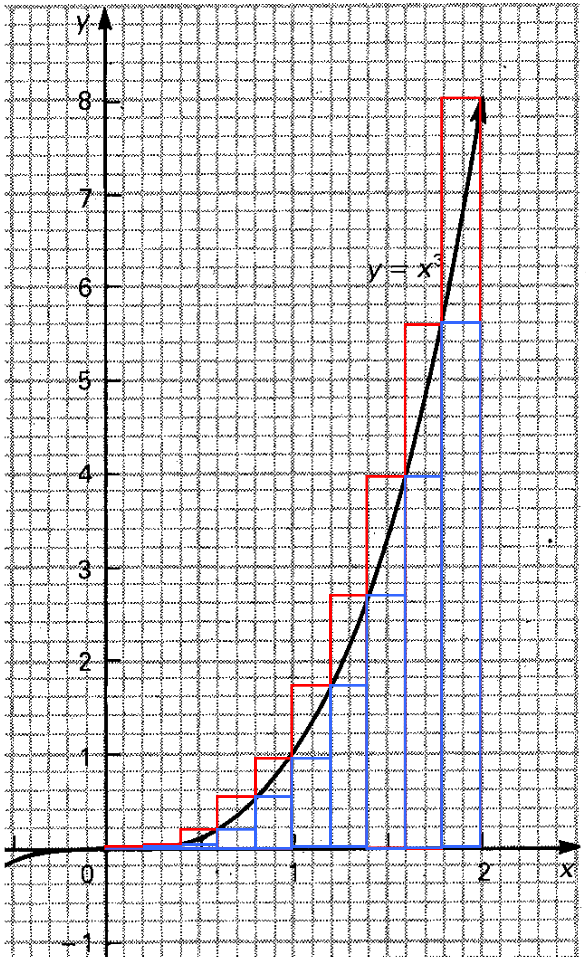
$$\left(\textit{Exact Area} = 4\ \textit{unit}^2\right)$$



$$0.4\{0^3+0.4^3+0.8^3+1.2^3+1.6^3\}\le \text{Area}\le 0.4\{0.4^3+0.8^3+1.2^3+1.6^3+2^3\}$$

$$2.56\le \text{Area}\le 5.76$$

$$\text{Estimate Area}=4.16\,\text{unit}^2$$



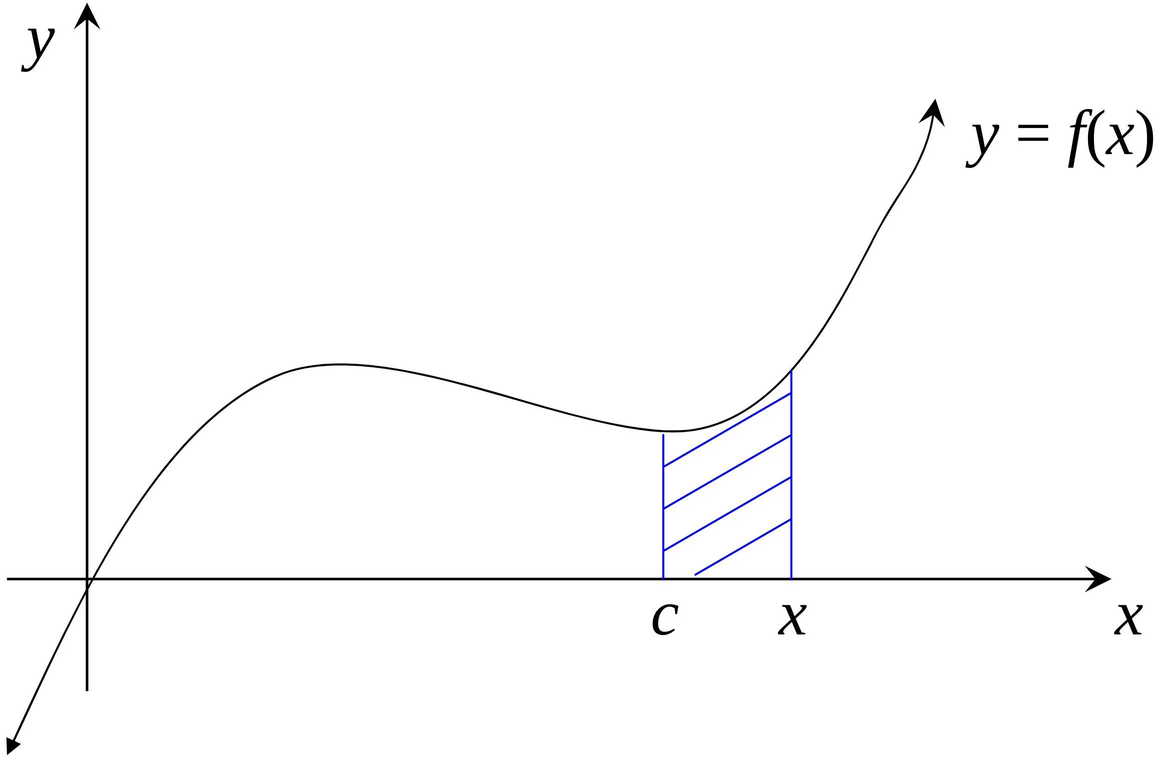
$$0.2\{0^3+0.2^3+0.4^3+0.6^3+0.8^3+1^3+1.2^3+1.4^3+1.6^3+1.8^3\}\le$$

$$\text{Area}\le 0.2\{0.2^3+0.4^3+0.6^3+0.8^3+1^3+1.2^3+1.4^3+1.6^3+1.8^3+2^3\}$$

$$3.24\le \text{Area}\le 4.84$$

$$\text{Estimate Area} = 4.04\ \text{unit}^2$$

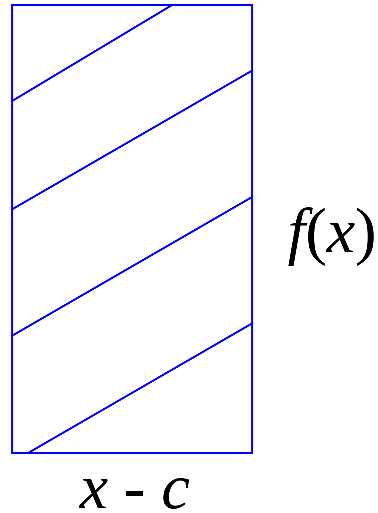
As the widths decrease, the estimate becomes more accurate, lets investigate **one** of these rectangles.



$A(c)$ is the area from 0 to c

$A(x)$ is the area from 0 to x

∴ ***A*(*x*) – *A*(*c*)** denotes the area from *c* to *x*, and can be estimated by the rectangle;



$$A(x) - A(c) \approx (x - c)\,f(x)$$

$$f(x) \approx \frac{A(x) - A(c)}{x - c}$$

$$= \frac{A(c+h) - A(c)}{h}$$

h = width of rectangle

As the width of the rectangle decreases, the estimate becomes more accurate.

$$\text{i.e. as } h \to 0, \text{ the Area becomes exact}$$

$$f(x) = \lim_{h \to 0} \frac{A(c+h) - A(c)}{h}$$

$$= \lim_{h \to 0} \frac{A(x+h) - A(x)}{h} \qquad (\because \text{as } h \to 0, c \to x)$$

$$= A'(x)$$

$\therefore$ the equation of the curve is the derivative of the Area function.

The area under the curve $y = f(x)$ between $x = a$ and $x = b$ is;

$$A = \int_a^b f(x)dx$$

$$= F(b) - F(a)$$

where $F(x)$ is the primitive function of $f(x)$

e.g. (i) Find the area under the curve $y = x^3$, between $x = 0$ and $x= 2$

$$A = \int_0^2 x^3 dx$$

$$= \left[\frac{1}{4}x^4\right]_0^2$$

$$= \frac{1}{4}\left\{2^4 - 0^4\right\}$$

$$= 4 \text{ units}^2$$

$$(ii)\int_2^3 \left(x^2 + 1\right)dx = \left[\frac{1}{3}x^3 + x\right]_2^3$$

$$= \left\{\frac{1}{3}(3)^3 + 3\right\} - \left\{\frac{1}{3}(2)^3 + 2\right\}$$

$$= \frac{22}{3}$$

$$(iii)\int_4^5 x^{-3}dx = \left[-\frac{1}{2}x^{-2}\right]_4^5$$

$$= -\frac{1}{2}\left\{\frac{1}{5^2}-\frac{1}{4^2}\right\}$$

$$= \frac{9}{800}$$

Exercise 11A; 1

Exercise 11B; 1 aefhi, 2ab (i,ii), 3ace, 4b, 5a, 7*