Polynomial Functions

A real polynomial $P(x)$ of degree n is an expression of the form;

$$P(x) = p_0 + p_1 x + p_2 x^2 + \ldots + p_{n-1} x^{n-1} + p_n x^n$$

$$\text{where: } p_n \neq 0$$

$$n \geq 0 \text{ and is an integer}$$

coefficients: $p_0, p_1, p_2, \cdots, p_n$

index (exponent): the powers of the pronumerals.

degree (order): the highest index of the polynomial. The polynomial is called **"polynomial of degree *n*"**

leading term: $p_n x^n$

leading coefficient: p_n

monic polynomial: leading coefficient is equal to one.

$P(x) = 0$: polynomial equation

$y = P(x)$: polynomial function

roots: solutions to the polynomial equation $P(x) = 0$

zeros: the values of x that make polynomial $P(x)$ zero. i.e. the x intercepts of the graph of the polynomial.

e.g. (i) Which of the following are polynomials?

a) $5x^3 - 7x^{\frac{1}{2}} - 2$ **NO**, can't have fraction as a power

b) $\dfrac{4}{x^2+3}$ **NO**, can't have negative as a power $4\left(x^2+3\right)^{-1}$

c) $\dfrac{x^2+3}{4}$ **YES**, $\dfrac{1}{4}x^2 + \dfrac{3}{4}$

d) 7 **YES**, $7x^0$

Exercise 4A; 1, 2acehi, 3bdf, 6bdf, 7, 9d, 10ad, 13

(ii) Determine whether $P(x) = x^3\left(8x+1\right) + 7x - 11 - \left(2x^2+1\right)\left(4x^2-3\right)$ is monic and state its degree.

$$P(x) = 8x^4 + x^3 + 7x - 11 - 8x^4 + 6x^2 - 4x^2 + 3$$
$$= x^3 + 2x^2 + 7x - 8$$

$\therefore$ monic, degree = 3