Properties Of Definite Integral

$$(1) \int_a^b x^n \, dx = \left[\frac{x^{n+1}}{n+1} \right]_a^b$$

$$(2) \int_a^b k f(x) \, dx = k \int_a^b f(x) \, dx \qquad \text{(can only factorise constants)}$$

$$(3) \int_a^b \{ f(x) \pm g(x) \} \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx$$

$$(4) \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx$$

$(5) \displaystyle\int_a^b x^n dx > 0 \qquad \text{,if } f(x) > 0 \text{ for } a < x < b$

$\qquad\qquad\qquad < 0 \qquad \text{,if } f(x) < 0 \text{ for } a < x < b$

$(6) \displaystyle\int_a^b f(x)dx < \int_a^b g(x)dx \qquad \text{,if } f(x) < g(x) \text{ for } a < x < b$

$(7) \displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$

$(8) \displaystyle\int_{-a}^a f(x) = 0, \qquad \text{if } f(x) \text{ is odd}$

$(9) \displaystyle\int_{-a}^a f(x) = 2\int_0^a f(x), \qquad \text{if } f(x) \text{ is even}$

$NOTE:$

$odd \times odd = even$

$odd \times even = odd$

$even \times even = even$

e.g. (i) $\displaystyle\int_1^2 6x^2\,dx$ $\quad = 6\left[\dfrac{1}{3}x^3\right]_1^2$

$\qquad\qquad\qquad\quad = 2\left(2^3 - 1^3\right)$

$\qquad\qquad\qquad\quad = 14$

$(ii)\displaystyle\int_0^5 \sqrt[3]{x}\,dx$ $\quad = \displaystyle\int_0^5 x^{\frac{1}{3}}\,dx$

$\qquad\qquad\qquad\quad = \dfrac{3}{4}\left[x^{\frac{4}{3}}\right]_0^5$

$\qquad\qquad\qquad\quad = \dfrac{3}{4}\left[x\sqrt[3]{x}\right]_0^5$

$\qquad\qquad\qquad\quad = \dfrac{3}{4}\left(5\sqrt[3]{5} - 0\right)$

$\qquad\qquad\qquad\quad = \dfrac{15\sqrt[3]{5}}{4}$

$$(iii) \int_{-2}^{2} \sin^5 x \, dx \ = 0 \qquad \qquad (\text{odd function})^5 = \text{odd function}$$

$$(iv) \int_{-1}^{1} (x^3 + 2x^2 + x + 1) \, dx \ = 2 \int_{0}^{1} (2x^2 + 1) \, dx$$

$$= 2 \left[\frac{2}{3} x^3 + x \right]_{0}^{1}$$

$$= 2 \left\{ \frac{2}{3} (1)^3 + 1 \right\} - 0$$

$$= \frac{10}{3}$$

Exercise 11C; 1bce, 2adf, 3ab (i, iii), 4bcf, 5, 6ac, 7df, 8b, 12b, 13*