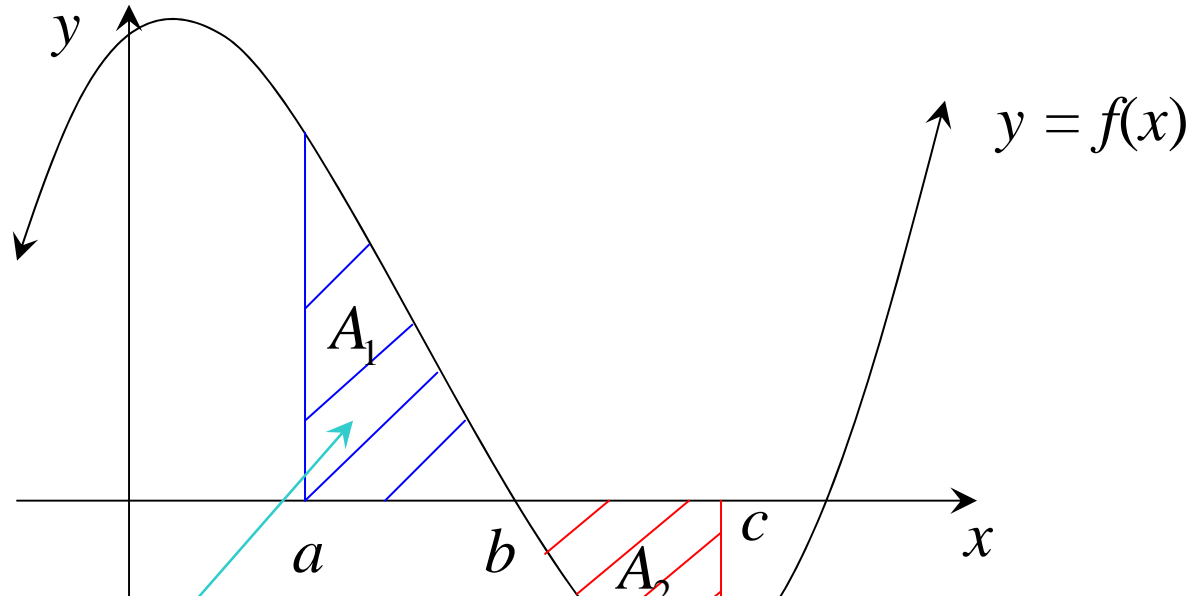


Areas

(1) Area Below x axis



$$\int_a^b f(x) dx > 0$$

$$\therefore A_1 = \int_a^b f(x) dx$$

$$\int_b^c f(x) dx < 0$$

$$\therefore A_2 = -\int_b^c f(x) dx$$

Area below x axis is given by;

$$A = -\int_b^c f(x)dx$$

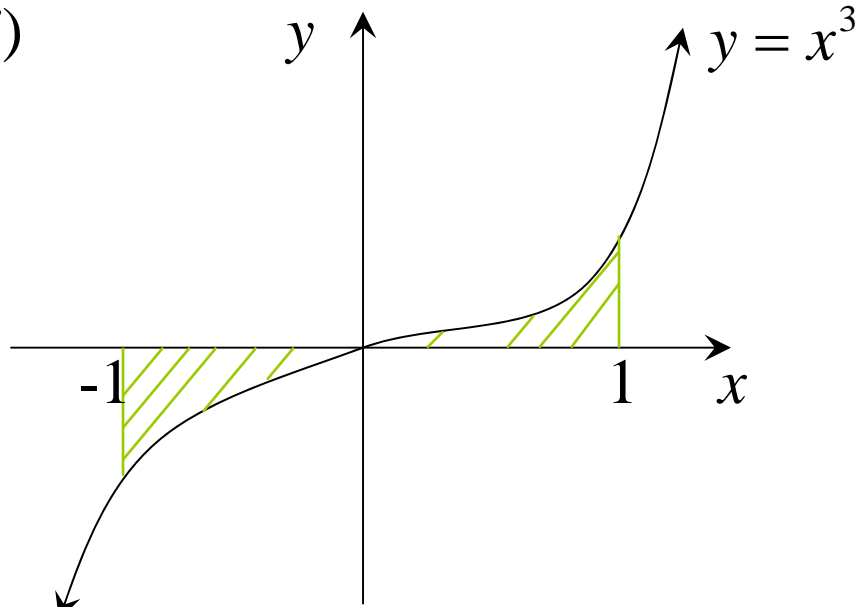
OR

$$= \left| \int_b^c f(x)dx \right|$$

OR

$$= \int_c^b f(x)dx$$

e.g. (i)

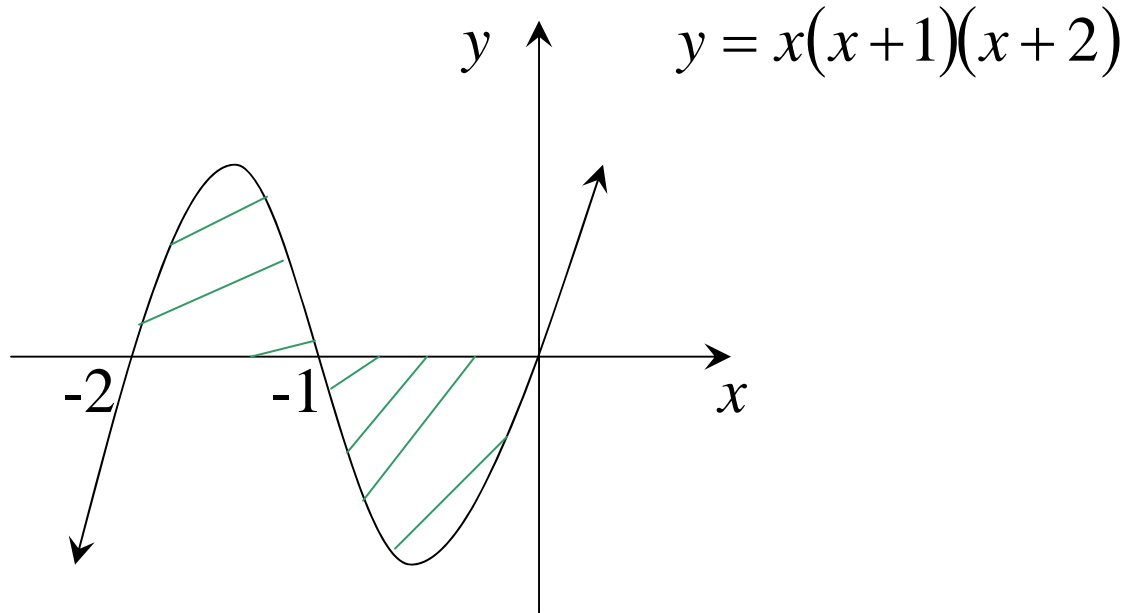


OR using symmetry of odd function

$$\begin{aligned} A &= 2 \int_0^1 x^3 dx \\ &= \frac{1}{2} [x^4]_0^1 \\ &= \frac{1}{2} \{1^4 - 0\} \\ &= \frac{1}{2} \text{ units}^2 \end{aligned}$$

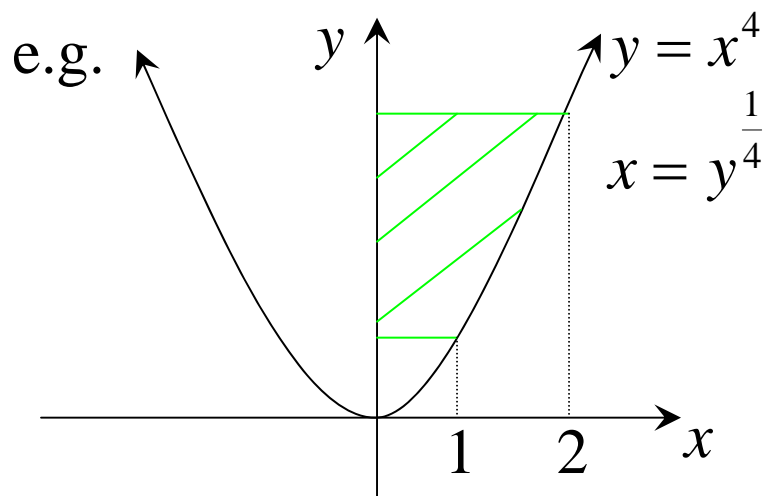
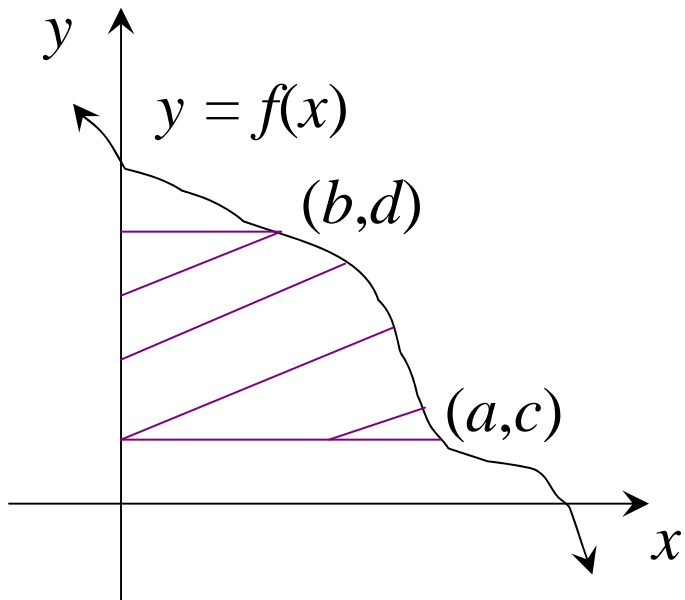
$$\begin{aligned} A &= -\int_{-1}^0 x^3 dx + \int_0^1 x^3 dx \\ &= -\frac{1}{4} [x^4]_{-1}^0 + \frac{1}{4} [x^4]_0^1 \\ &= -\frac{1}{4} \{0 - (-1)^4\} + \frac{1}{4} \{1^4 - 0\} \\ &= \frac{1}{4} + \frac{1}{4} \\ &= \frac{1}{2} \text{ units}^2 \end{aligned}$$

(ii)



$$\begin{aligned} A &= \int_{-2}^{-1} (x^3 + 3x^2 + 2x) dx - \int_{-1}^0 (x^3 + 3x^2 + 2x) dx \\ &= \left[\frac{1}{4}x^4 + x^3 + x^2 \right]_{-2}^{-1} + \left[\frac{1}{4}x^4 + x^3 + x^2 \right]_0^{-1} \\ &= 2 \left\{ \frac{1}{4}(-1)^4 + (-1)^3 + (-1)^2 \right\} - \left\{ \frac{1}{4}(-2)^4 + (-2)^3 + (-2)^2 \right\} - 0 \\ &= \underline{\underline{\frac{1}{2} \text{ units}^2}} \end{aligned}$$

(2) Area On The y axis



(1) Make x the subject

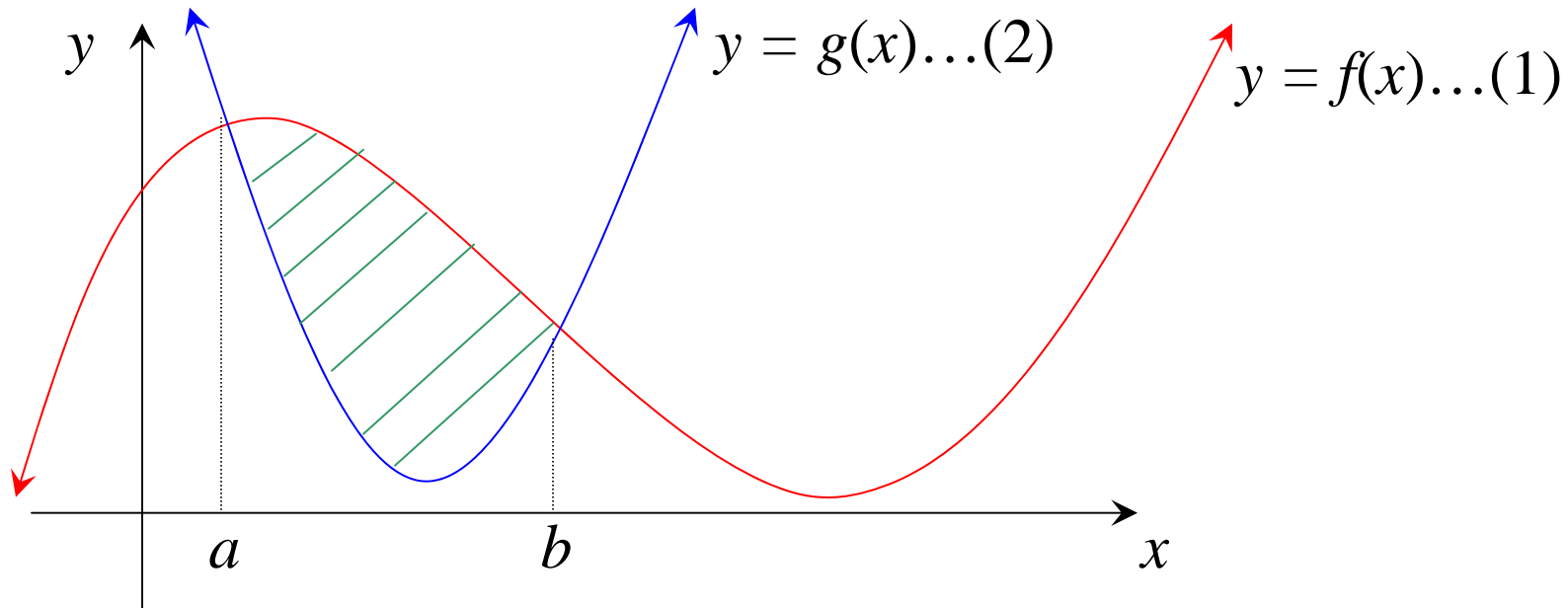
i.e. $x = g(y)$

(2) Substitute the y coordinates

$$(3) A = \int_c^d g(y) dy$$

$$\begin{aligned} A &= \int_1^{16} y^{\frac{1}{4}} dy \\ &= \frac{4}{5} \left[y^{\frac{5}{4}} \right]_1^{16} \\ &= \frac{4}{5} \left\{ 16^{\frac{5}{4}} - 1^{\frac{5}{4}} \right\} \\ &= \frac{124}{5} \text{ units}^2 \end{aligned}$$

(3) Area Between Two Curves

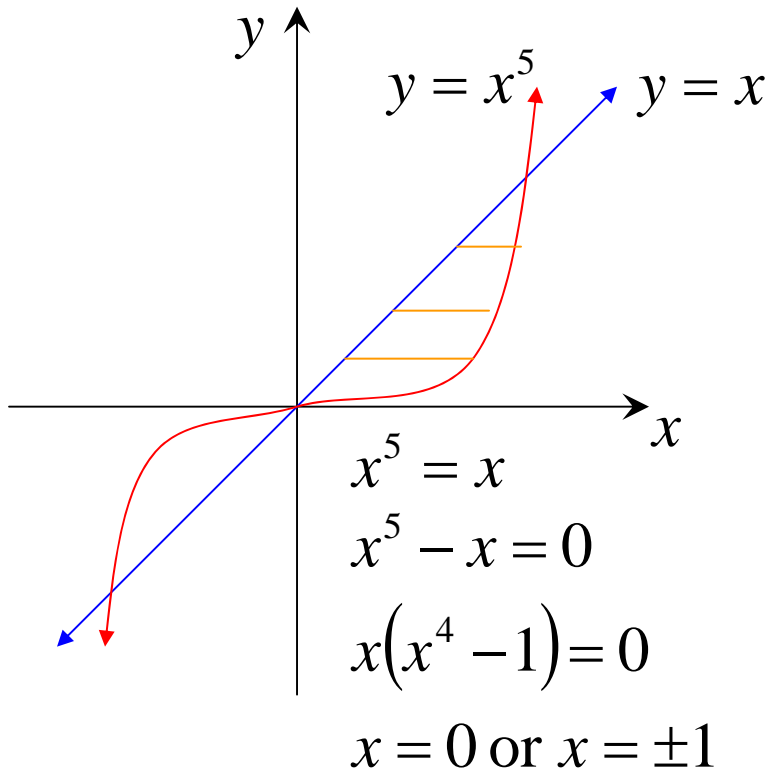


Area = Area under (1) – Area under (2)

$$= \int_a^b f(x)dx - \int_a^b g(x)dx$$

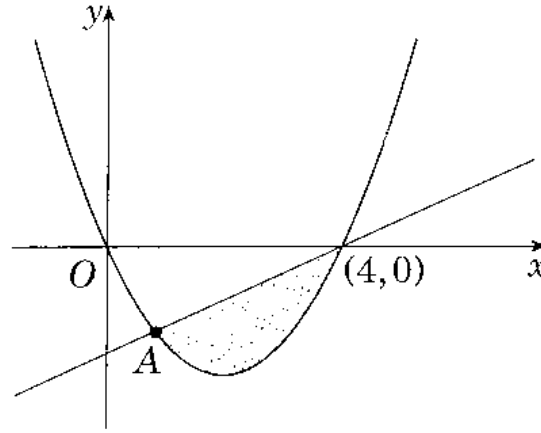
$$= \int_a^b (f(x) - g(x))dx$$

e.g. Find the area enclosed between the curves $y = x^5$ and $y = x$ in the positive quadrant.



$$\begin{aligned} A &= \int_0^1 (x - x^5) dx \\ &= \left[\frac{1}{2} x^2 - \frac{1}{6} x^6 \right]_0^1 \\ &= \left\{ \frac{1}{2} (1)^2 - \frac{1}{6} (1)^6 \right\} - 0 \\ &= \frac{1}{3} \text{ unit}^2 \end{aligned}$$

2002 HSC Question 4d)



The graphs of $y = x - 4$ and $y = x^2 - 4x$ intersect at the points $(4, 0)$ A.

(i) Find the coordinates of A (2)

To find points of intersection, solve simultaneously

$$x - 4 = x^2 - 4x$$

$$x^2 - 5x + 4 = 0$$

$$(x - 4)(x - 1) = 0$$

$$x = 1 \quad \text{or} \quad x = 4$$

$$\therefore \underline{A \text{ is } (1, -3)}$$

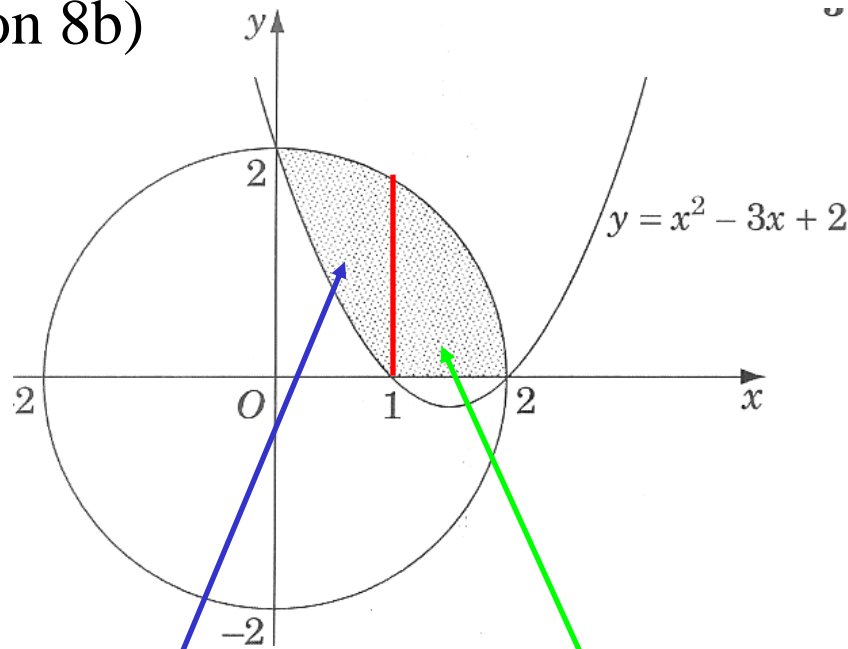
(ii) Find the area of the shaded region bounded by $y = x^2 - 4x$ and (3)

$$y = x - 4.$$

$$\begin{aligned} A &= \int_1^4 (x - 4 - (x^2 - 4x)) dx \\ &= \int_1^4 (-x^2 + 5x - 4) dx \\ &= \left[-\frac{1}{3}x^3 + \frac{5}{2}x^2 - 4x \right]_1^4 \\ &= -\frac{1}{3}(4)^3 + \frac{5}{2}(4)^2 - 4(4) - \left\{ -\frac{1}{3}(1)^3 + \frac{5}{2}(1)^2 - 4(1) \right\} \\ &= \underline{\underline{\frac{9}{2} \text{ units}^2}} \end{aligned}$$

2005 HSC Question 8b)

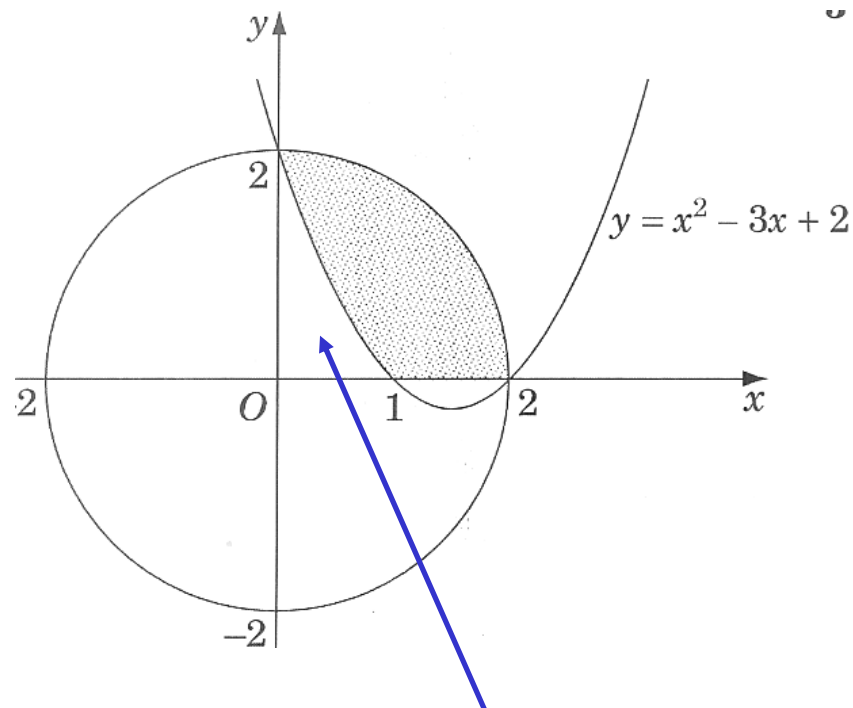
(3)



The shaded region in the diagram is bounded by the circle of radius 2 centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x axis. By considering the difference of two areas, find the area of the shaded region.

Note: area must be broken up into two areas, due to the different boundaries.

Area between circle and parabola and area between circle and x axis



It is easier to subtract the area under the parabola from the quadrant.

$$\begin{aligned}
 A &= \frac{1}{4}\pi(2)^2 - \int_0^1 (x^2 - 3x + 2)dx \\
 &= \pi - \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x \right]_0^1 \\
 &= \pi - \left\{ \frac{1}{3}(1)^3 - \frac{3}{2}(1)^2 + 2(1) - 0 \right\} \\
 &= \left(\pi - \frac{5}{6} \right) \text{units}^2
 \end{aligned}$$

Exercise 11E; 2bceh, 3bd, 4bd, 5bd, 7begj, 8d, 9a, 11, 18*

Exercise 11F; 1bdeh, 4bd, 7d, 10, 11b, 13, 15*