Polynomial Results

1. If $P(x)$ has k distinct real zeros, $a_1, a_2, a_3, \ldots, a_k$, then; $(x-a_1)(x-a_2)(x-a_3)\ldots(x-a_k)$ is a factor of $P(x)$.

e.g. Show that 1 and –2 are zeros of $P(x) = x^4 + x^3 + 3x^2 + 5x - 10$ and hence factorise $P(x)$.

$$P(1) = (1)^4 + (1)^3 + 3(1)^2 + 5(1) - 10$$

$$= 0$$

$$P(-2) = (-2)^4 + (2)^3 + 3(-2)^2 + 5(-2) - 10$$

$$= 0$$

$\therefore 1, -2$ are zeros of $P(x)$ and $(x-1)(x+2)$ is a factor

$$P(x) = x^4 + x^3 + 3x^2 + 5x - 10$$

$$= (x^2 + x - 2)(x^2 \qquad +5)$$

$$= (x-1)(x+2)(x^2+5)$$

2. If $P(x)$ has degree n and has n distinct real zeros, $a_1, a_2, a_3, \ldots, a_n$, then $P(x) = (x - a_1)(x - a_2)(x - a_3)\ldots(x - a_n)$

3. A polynomial of degree n cannot have more than n distinct real zeros

e.g. The polynomial $P(x)$ has a double zero at –7 and a single zero at 2. Write down;

a) a possible polynomial

$$P(x) = k(x-2)(x+7)^2 Q(x)$$

where - $Q(x)$ is a polynomial and does not have a zero at 2 or –7

- k is a real number

b) a monic polynomial of degree 3.

$$P(x) = (x-2)(x+7)^2$$

c) A monic polynomial of degree 4

$$P(x) = (x-2)(x+7)^2(x-a)$$

where $a \neq 2$ or -7

d) a polynomial of degree 5.

$$P(x) = k(x-2)(x+7)^2 Q(x)$$

where - $Q(x)$ is a polynomial of degree 2,
and does not have a zero at 2 or –7

- k is a real number

4. If $P(x)$ has degree n and has **more** than n real zeros, then $P(x)$ is the zero polynomial. i.e. $P(x) = 0$ for all values of x

Exercise 4E; 1, 3bd, 4a, 5b, 6a, 8, 12ad