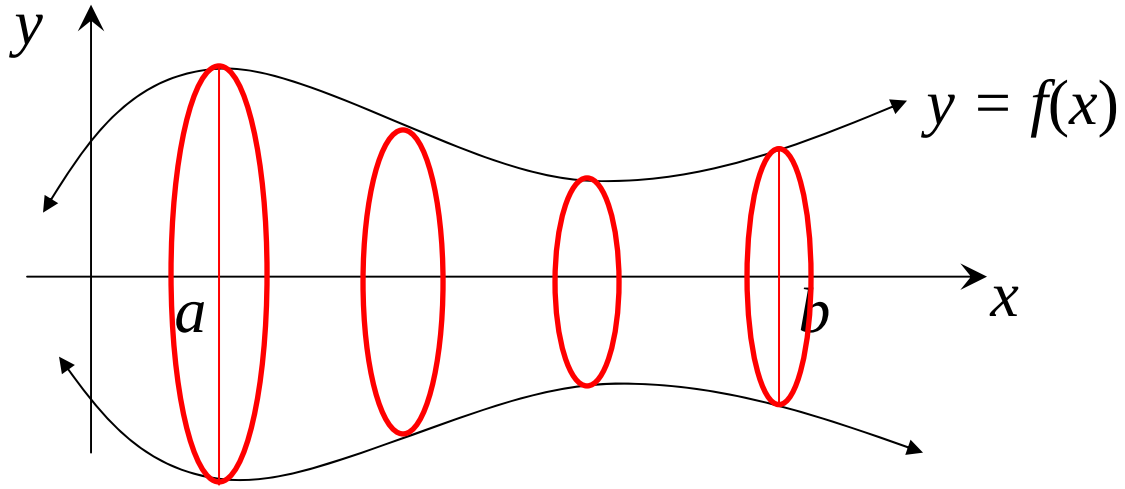
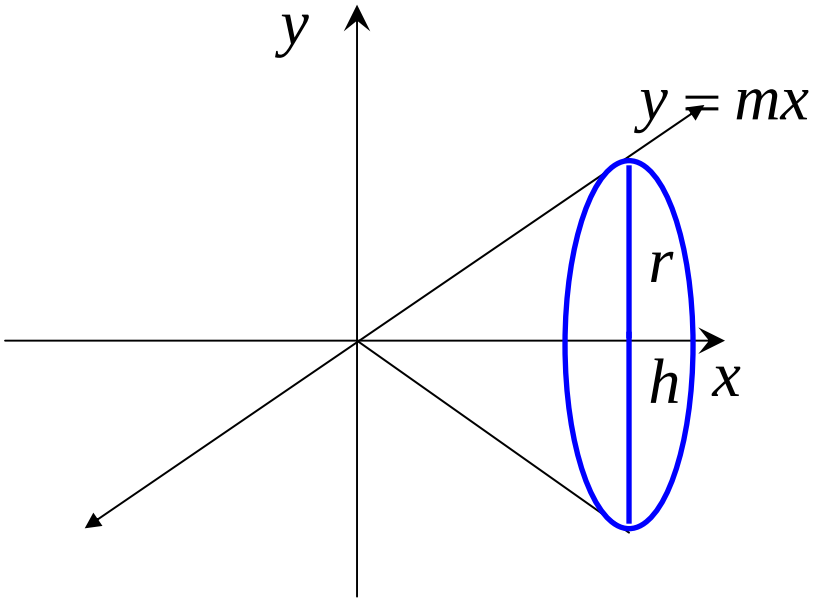
Volumes of Solids of Revolution



Volume of a solid of revolution about; $(i) x \text{ axis} : V = \pi \int_a^b y^2 dx$

$(ii) y \text{ axis} : V = \pi \int_c^d x^2 dy$

e.g. (*i*) <u>cone</u>



$$m = \frac{r}{h}$$

$$V = \pi \int y^2 dx$$

$$= \pi \int_0^h m^2 x^2 dx$$

$$= \pi m^2 \left[\frac{1}{3} x^3 \right]_0^h$$

$$= \frac{1}{3} \pi m^2 \left(h^3 - 0 \right)$$

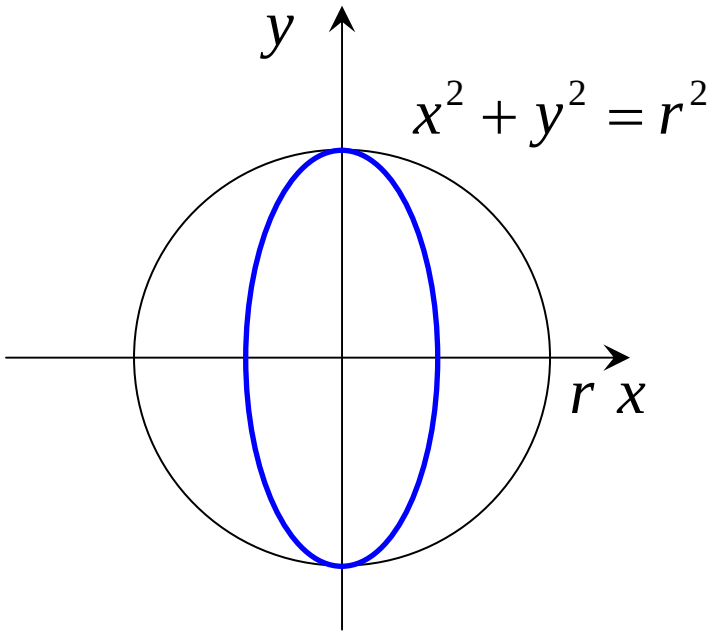
$$= \frac{1}{3} \pi m^2 h^3$$

$$= \frac{1}{3} \pi \left(\frac{r}{h} \right)^2 h^3$$

$$= \frac{\pi r^2 h^3}{3h^2}$$

$$= \frac{\pi r^2 h}{3}$$

(*ii*) sphere



$$V = \pi \int y^2 dx$$

$$= 2\pi \int_0^r \left(r^2 - x^2\right) dx$$

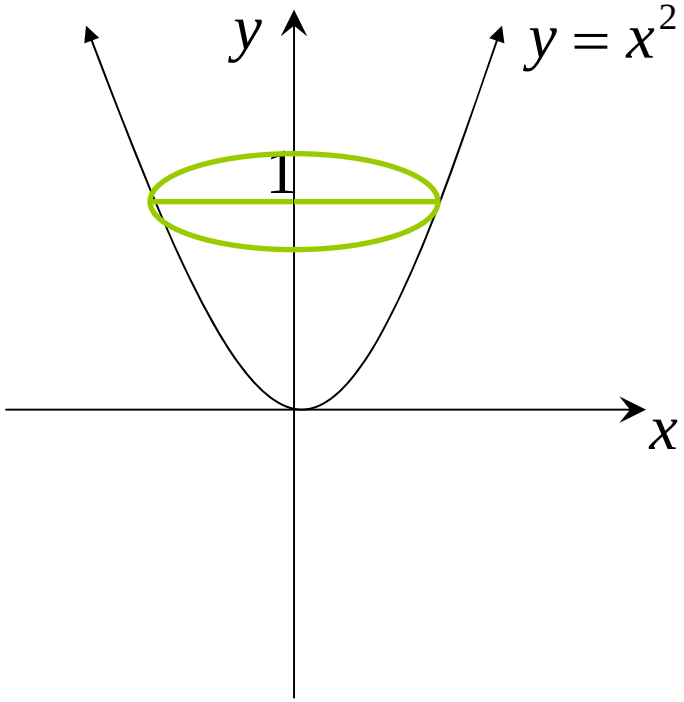
$$= 2\pi \left[r^2 x - \frac{1}{3} x^3 \right]_0^r$$

$$= 2\pi \left\{ \left(r^2 (r) - \frac{1}{3} r^3 \right) - 0 \right\}$$

$$= 2\pi \left(\frac{2}{3} r^3 \right)$$

$$= \frac{4}{3} \pi r^3$$

(*iii*) Find the volume of the solid generated when $y = x^2$ is revolved around the y axis between $y = 0$ and $y = 1$.



$$V = \pi \int x^2 \, dy$$

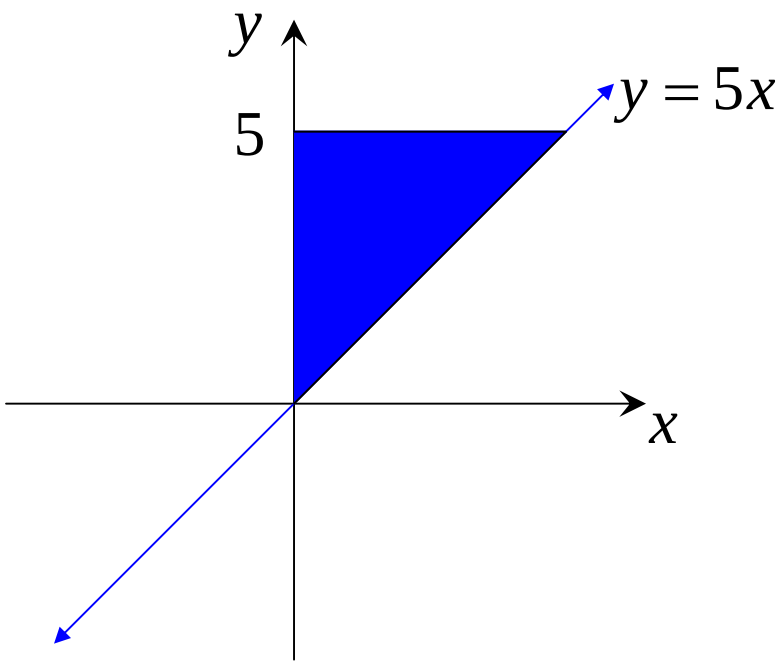
$$= \pi \int_0^1 y \, dy$$

$$= \frac{\pi}{2} \left[y^2 \right]_0^1$$

$$= \frac{\pi}{2} \left(1^2 - 0 \right)$$

$$= \frac{\pi}{2} \text{ units}^3$$

(*iv*) Find the volume of the solid when the shaded region is rotated about the x axis



$$V = \pi \int y^2 dx$$

$$= \pi \int_0^1 \left\{5^2 - (5x)^2\right\} dx$$

$$= \pi \int_0^1 \left(25 - 25x^2\right) dx$$

$$= 25\pi \left[x - \frac{1}{3}x^3 \right]_0^1$$

$$= 25\pi \left\{ 1 - \frac{1}{3}(1)^3 - 0 \right\}$$

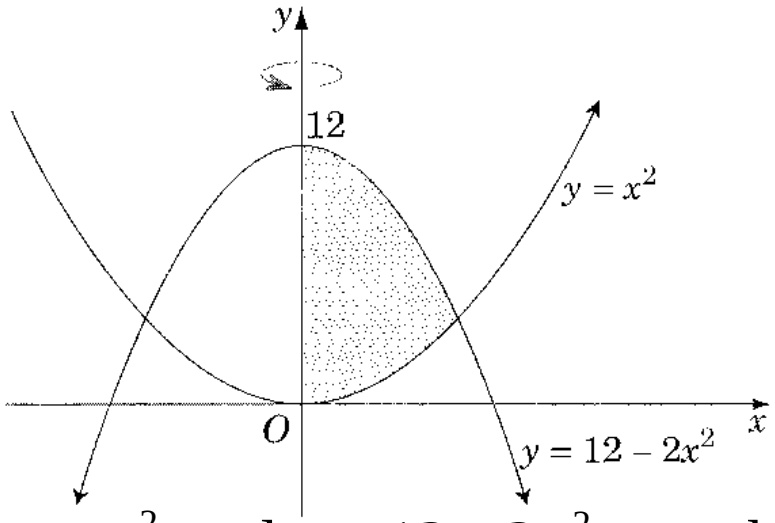
$$= \frac{50\pi}{3} \text{ unit}^3$$

OR $V = \pi r^2 h - \frac{1}{3}\pi r^2 h$

$$= \frac{2}{3}\pi (5)^2 (1)$$

$$= \frac{50\pi}{3} \text{ units}^3$$

2005 HSC Question 6c)



The graphs of the curves $y = x^2$ and $y = 12 - 2x^2$ are shown in the diagram.

(i) Find the points of intersection of the two curves. **(1)**

$$x^2 = 12 - 2x^2$$

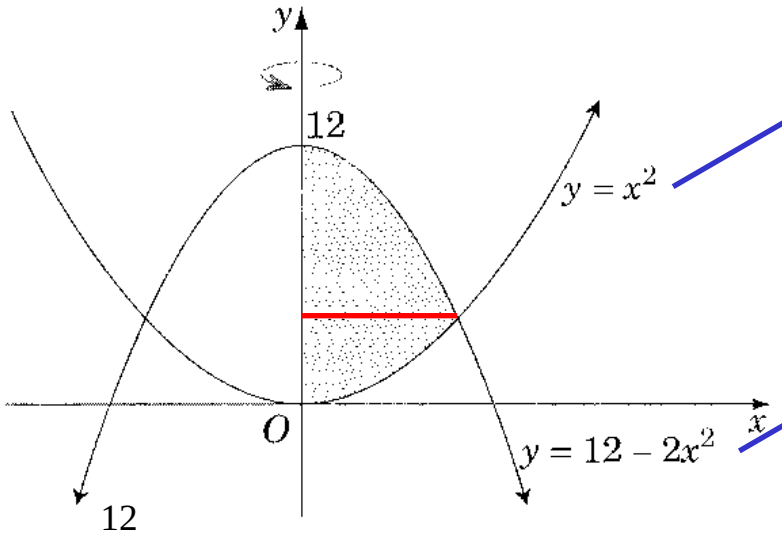
$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

$\therefore$ meet at $(2, 4)$

(ii) The shaded region between the two curves and the y axis is rotated about the y axis. By splitting the shaded region into two parts, or otherwise, find the volume of the solid formed. **(3)**



$x^2 = y$

$x^2 = 6 - \frac{1}{2}y$

$$V = \pi \int x^2 dy$$

$$V = \pi \int_0^4 \left(6 - \frac{1}{2}y\right) dy + \pi \int_4^{12} y dy$$

$$= \pi \left[6y - \frac{1}{4}y^2\right]_0^4 + \pi \left[\frac{1}{2}y^2\right]_4^{12}$$

$$= \pi \left\{6(4) - \frac{1}{4}(4)^2 - 0\right\} + \frac{\pi}{2}\left\{(12)^2 - (4)^2\right\}$$

$$= 84\pi \text{ units}^3$$

Exercise 11G; 3bdef, 4eg, 5cd, 6d, 8ad,
12, 16a, 19 to 22, 24*, 25*