Roots and Coefficients

Quadratics $ax^2 + bx + c = 0$

$$\alpha + \beta = -\frac{b}{a} \qquad \alpha\beta = \frac{c}{a}$$

Cubics $ax^3 + bx^2 + cx + d = 0$

$$\alpha + \beta + \gamma = -\frac{b}{a} \qquad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

Quartics $ax^4 + bx^3 + cx^2 + dx + e = 0$

$$\alpha + \beta + \gamma + \delta = -\frac{b}{a} \qquad \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$$

$$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a} \qquad \alpha\beta\gamma\delta = \frac{e}{a}$$

For the polynomial equation;

$$ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \cdots = 0$$

$$\sum \alpha = -\frac{b}{a}$$ (sum of roots, one at a time)

$$\sum \alpha\beta = \frac{c}{a}$$ (sum of roots, two at a time)

$$\sum \alpha\beta\gamma = -\frac{d}{a}$$ (sum of roots, three at a time)

$$\sum \alpha\beta\gamma\delta = \frac{e}{a}$$ (sum of roots, four at a time)

Note:

$$\sum \alpha^2 = \left(\sum \alpha\right)^2 - 2\sum \alpha\beta$$

e.g. (i) If α, β and γ are the roots of $2x^3 - 5x^2 - 3x + 1 = 0,$ find the values of;

a) $4\alpha + 4\beta + 4\gamma - 7\alpha\beta\gamma$

$$\alpha + \beta + \gamma = \frac{5}{2} \qquad \alpha\beta + \alpha\gamma + \beta\gamma = -\frac{3}{2} \qquad \alpha\beta\gamma = -\frac{1}{2}$$

$$4\alpha + 4\beta + 4\gamma - 7\alpha\beta\gamma = 4\left(\frac{5}{2}\right) - 7\left(-\frac{1}{2}\right)$$

$$= \frac{27}{2}$$

b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$$= \frac{-\frac{3}{2}}{-\frac{1}{2}}$$

$$= -3$$

c) $\alpha^2 + \beta^2 + \gamma^2$

$$= (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$$

$$= \left(\frac{5}{2}\right)^2 - 2\left(-\frac{3}{2}\right)$$

$$= \frac{37}{4}$$

1988 Extension 1 HSC Q2c)

If α, β and γ are the roots of $x^3 - 3x + 1 = 0$ find:

(i) $\alpha + \beta + \gamma$

$$\alpha + \beta + \gamma = 0$$

(ii) $\alpha\beta\gamma$

$$\alpha\beta\gamma = -1$$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma}$$

$$= \frac{-3}{-1}$$

$$= 3$$

2003 Extension 1 HSC Q4c)

It is known that two of the roots of the equation $2x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other.
Find the value of k.

Let the roots be $\alpha, \dfrac{1}{\alpha}$ and β

$$(\alpha)\left(\frac{1}{\alpha}\right)(\beta) = \frac{-6}{2}$$

$$\beta = -3$$

$$P(-3) = 0$$

$$2(-3)^3 + (-3)^2 - k(-3) + 6 = 0$$

$$-54 + 9 + 3k + 6 = 0$$

$$3k = 39$$

$$\underline{k = 13}$$

The cubic polynomial $P(x) = x^3 + rx^2 + sx + t$, where *r*, *s* and *t* are real numbers, has three real zeros, 1, α and $-\alpha$

(i) Find the value of *r*

$$1 + \alpha + -\alpha = -r$$

$$\underline{r = -1}$$

(ii) Find the value of *s* + *t*

$$(1)(\alpha) + (1)(-\alpha) + (\alpha)(-\alpha) = s$$

$$s = -\alpha^2$$

$$(1)(\alpha)(-\alpha) = -t$$

$$t = \alpha^2$$

$$\underline{\therefore s + t = 0}$$

Exercise 4F; 2, 4, 5ac, 6ac, 8, 10a, 13, 15, 16ad, 17, 18, 19