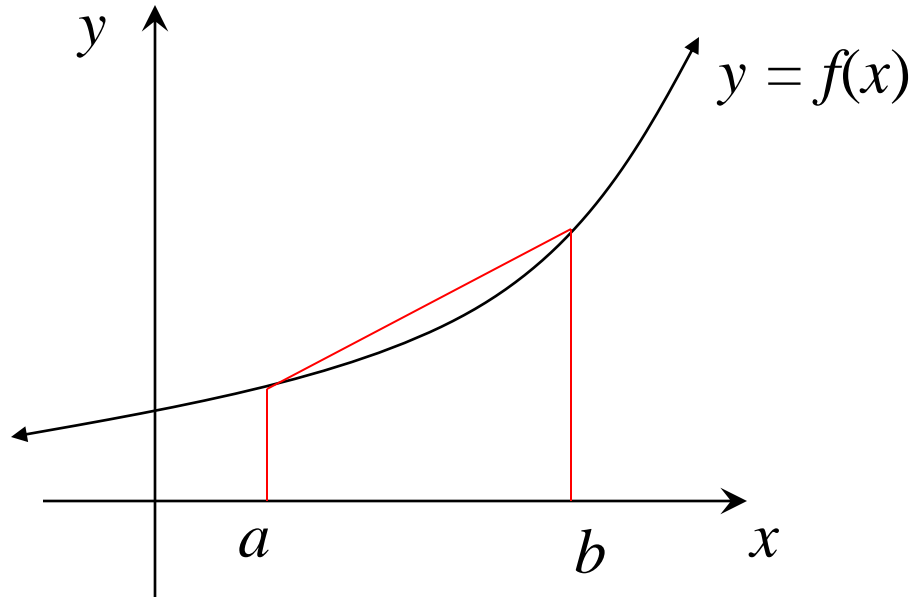


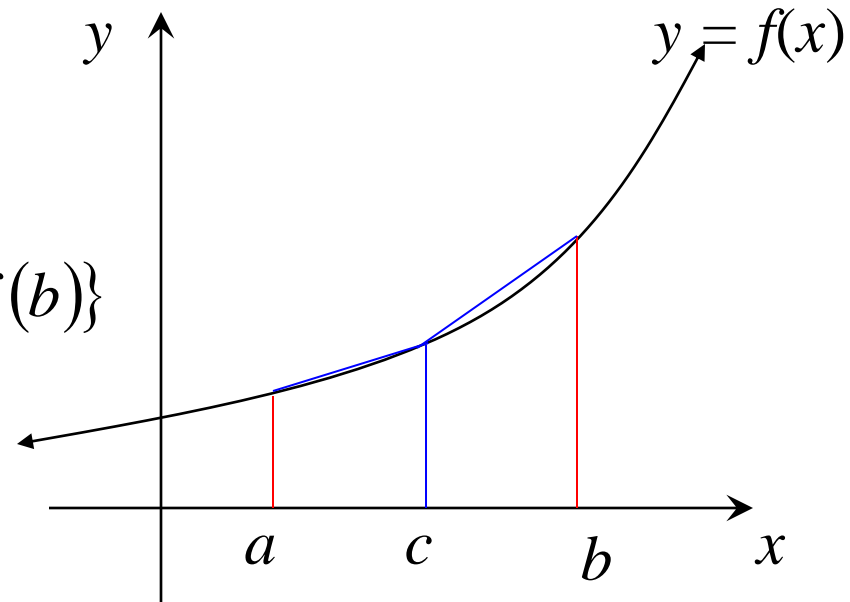
# *Approximations To Areas*

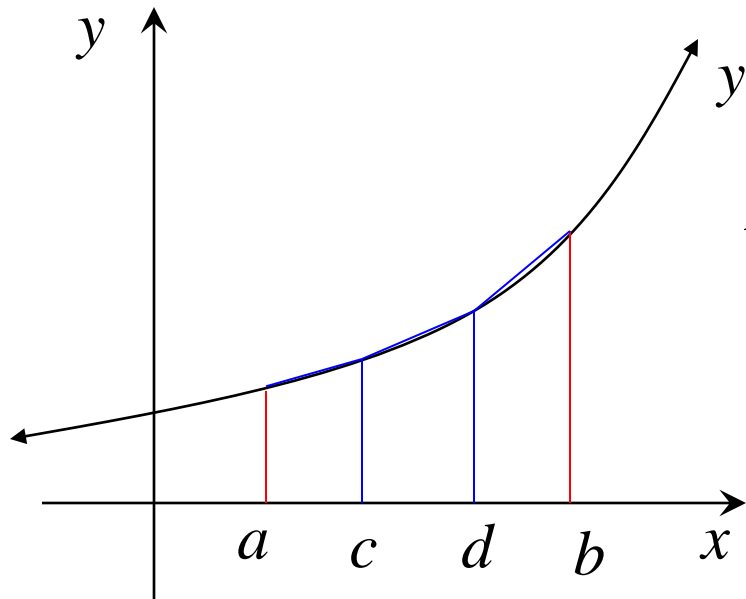
## (1) Trapezoidal Rule



$$A \approx \frac{b-a}{2} \{f(a) + f(b)\}$$

$$\begin{aligned} A &\approx \frac{c-a}{2} \{f(a) + f(c)\} + \frac{b-c}{2} \{f(c) + f(b)\} \\ &= \frac{c-a}{2} \{f(a) + 2f(c) + f(b)\} \end{aligned}$$





$$\begin{aligned}
 A &\approx \frac{c-a}{2} \{f(a) + f(c)\} + \frac{d-c}{2} \{f(c) + f(d)\} \\
 &\quad + \frac{b-d}{2} \{f(d) + f(b)\} \\
 &= \frac{c-a}{2} \{f(a) + 2f(c) + 2f(d) + f(b)\}
 \end{aligned}$$

In general:

$$\begin{aligned}
 \text{Area} &= \int_a^b f(x) dx \\
 &\approx \frac{h}{2} \{y_0 + 2y_{\text{others}} + y_n\}
 \end{aligned}$$

where  $h = \frac{b-a}{n}$

$n$  = number of trapeziums

**NOTE:** there is always one more function value than interval

e.g. Use the Trapezoidal Rule with 4 intervals to estimate the

area under the curve  $y = (4 - x^2)^{\frac{1}{2}}$ , between  $x = 0$  and  $x = 2$   
(correct to 3 decimal points)

$$h = \frac{b - a}{n}$$

$$= \frac{2 - 0}{4}$$

$$= 0.5$$

	1	2	2	2	1
$x$	0	0.5	1	1.5	2
$y$	2	1.9365	1.7321	1.3229	0

$$\text{Area} \approx \frac{h}{2} \{y_0 + 2y_{\text{others}} + y_n\}$$

$$= \frac{0.5}{2} \{2 + 2(1.9365 + 1.7321 + 1.3229) + 0\}$$

$$= \underline{2.996 \text{ units}^2} \quad (\text{exact value} = \pi)$$

$$\% \text{ error} = \frac{3.142 - 2.996}{3.142} \times 100$$

$$= 4.6\%$$

## (2) Simpson's Rule

$$\text{Area} = \int_a^b f(x)dx$$

$$\approx \frac{h}{3} \{y_0 + 4y_{\text{odd}} + 2y_{\text{even}} + y_n\}$$

where  $h = \frac{b-a}{n}$

$n$  = number of intervals

e.g.

	1	4	2	4	1
$x$	0	0.5	1	1.5	2
$y$	2	1.9365	1.7321	1.3229	0

$$\text{Area} \approx \frac{h}{3} \{y_0 + 4y_{\text{odd}} + 2y_{\text{even}} + y_n\}$$

$$= \frac{0.5}{3} \{2 + 4(1.9365 + 1.3229) + 2(1.7321) + 0\}$$

$$= \underline{3.084 \text{ units}^2}$$

$$\% \text{ error} = \frac{3.142 - 3.084}{3.142} \times 100$$

$$= 1.8\%$$

# *Alternative working out!!!*

## (1) Trapezoidal Rule

	1	2	2	2	1
$x$	0	0.5	1	1.5	2
$y$	2	1.9365	1.7321	1.3229	0

$$\begin{aligned} \text{Area} &\approx \frac{2 + 2(1.9365 + 1.7321 + 1.3229) + 0}{1 + 2 + 2 + 2 + 1} \times (2 - 0) \\ &= \underline{2.996 \text{ units}^2} \end{aligned}$$

## (2) Simpson's Rule

	1	4	2	4	1
$x$	0	0.5	1	1.5	2
$y$	2	1.9365	1.7321	1.3229	0

$$\text{Area} \approx \frac{2 + 4(1.9365 + 1.3229) + 2(1.7321) + 0}{1 + 4 + 2 + 4 + 1} \times (2 - 0)$$
$$= 3.084 \text{ units}^2$$

**Exercise 11I; odds**

**Exercise 11J; evens**