Mathematical Induction

<u>Step 1</u>: Prove the result is true for $n = 1$ (or whatever the first term is)

<u>Step 2</u>: Assume the result is true for $n = k$, where k is a positive integer (or another condition that matches the question)

<u>Step 3</u>: Prove the result is true for $n = k + 1$

NOTE: It is important to note in your conclusion that the result is true for $n = k + 1$ **if it is true for $\boldsymbol{n = k}$**

<u>Step 4</u>: Since the result is true for $n = 1$, then the result is true for all positive integral values of n by induction

$e.g.(i)\ 1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$

Step 1: Prove the result is true for $n = 1$

$$LHS = 1^2 = 1$$

$$RHS = \frac{1}{3}(1)(2-1)(2+1) = \frac{1}{3}(1)(1)(3) = 1$$

$\therefore LHS = RHS$

Hence the result is true for $n = 1$

Step 2: Assume the result is true for $n = k$, where k is a positive integer

$i.e.\ 1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{1}{3}k(2k-1)(2k+1)$

Step 3: Prove the result is true for $n = k + 1$

$i.e.$ Prove: $1^2+3^2+5^2+\ldots+(2k+1)^2=\frac{1}{3}(k+1)(2k+1)(2k+3)$

Proof:

$$1^2+3^2+5^2+\ldots+(2k+1)^2$$

$$=\underbrace{1^2+3^2+5^2+\ldots+(2k-1)^2}_{S_k}+\underbrace{(2k+1)^2}_{T_{k+1}}$$

$$=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$$

$$=(2k+1)\left[\frac{1}{3}k(2k-1)+(2k+1)\right]$$

$$=\frac{1}{3}(2k+1)[k(2k-1)+3(2k+1)]$$

$$=\frac{1}{3}(2k+1)(2k^2-k+6k+3)$$

$$=\frac{1}{3}(2k+1)(2k^2+5k+3)$$

$$=\frac{1}{3}(2k+1)(k+1)(2k+3)$$

Hence the result is true for $n = k + 1$ if it is also true for $n = k$

<u>Step 4</u><u>:</u> Since the result is true for $n = 1$, then the result is true for all positive integral values of n by induction

$$(ii)\ \sum_{k=1}^{n}\frac{1}{(2k-1)(2k+1)}=\frac{n}{2n+1}$$

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$

Step 1: Prove the result is true for $n = 1$

$$LHS=\frac{1}{1\times 3}=\frac{1}{3}$$

$$RHS=\frac{1}{2+1}=\frac{1}{3}$$

$$\therefore LHS = RHS$$

Hence the result is true for $n = 1$

Step 2: Assume the result is true for $n = k$, where k is a positive integer

$$i.e.\ \frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2k-1)(2k+1)}=\frac{k}{2k+1}$$

Step 3: Prove the result is true for $n = k + 1$

$$i.e.\ \text{Prove}: \frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2k+1)(2k+3)}=\frac{k+1}{2k+3}$$

Proof:

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2k+1)(2k+3)}$$

$$= \frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2k-1)(2k+1)}+\frac{1}{(2k+1)(2k+3)}$$

$$=\frac{k}{2k+1}+\frac{1}{(2k+1)(2k+3)}$$

$$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$=\frac{(2k+1)(k+1)}{(2k+1)(2k+3)}$$

$$= \frac{(k+1)}{(2k+3)}$$

Hence the result is true for $n = k + 1$ if it is also true for $n = k$

Step 4: Since the result is true for $n = 1$, then the result is true for all positive integral values of n by induction

Exercise 6N; 1 ace etc, 10(polygon), 13