Mathematical Induction

$e.g.(iii)$ Prove $n(n+1)(n+2)$ is divisible by 3

Step 1: Prove the result is true for $n = 1$

$$(1)(2)(3)$$

$= 6$ which is divisible by 3

Hence the result is true for $n = 1$

Step 2: Assume the result is true for $n = k$, where k is a positive integer

$i.e.\ k(k+1)(k+2) = 3P$, where P is an integer

Step 3: Prove the result is true for $n = k + 1$

$i.e.$ Prove: $(k+1)(k+2)(k+3) = 3Q$, where Q is an integer

Proof:

$$(k+1)(k+2)(k+3)$$
$$= k(k+1)(k+2) + 3(k+1)(k+2)$$
$$= 3P + 3(k+1)(k+2)$$
$$= 3[P + (k+1)(k+2)]$$
$$= 3Q, \text{ where } Q = P + (k+1)(k+2) \text{ which is an integer}$$

Hence the result is true for $n = k + 1$ if it is also true for $n = k$

Step 4: Since the result is true for $n = 1$, then the result is true for all positive integral values of n by induction.

(iv) Prove $3^{3n} + 2^{n+2}$ is divisible by 5

Step 1: Prove the result is true for $n = 1$

$$3^3 + 2^3$$

$$= 27 + 8$$

$= 35$ which is divisible by 5

Hence the result is true for $n = 1$

Step 2: Assume the result is true for $n = k$, where k is a positive integer

$i.e.\ 3^{3k} + 2^{k+2} = 5P$, where P is an integer

Step 3: Prove the result is true for $n = k + 1$

$i.e.$ Prove: $3^{3k+3} + 2^{k+3} = 5Q$, where Q is an integer

Proof: $3^{3k+3}+2^{k+3}$

$$= 27\cdot 3^{3k}+2^{k+3}$$
$$= 27\left(5P-2^{k+2}\right)+2^{k+3}$$
$$= 135P-27\cdot 2^{k+2}+2^{k+3}$$
$$= 135P-27\cdot 2^{k+2}+2\cdot 2^{k+2}$$
$$= 135P-25\cdot 2^{k+2}$$
$$= 5\left(27P-5\cdot 2^{k+2}\right)$$

$= 5Q$, where $Q = 27P-5\cdot 2^{k+2}$ which is an integer

Hence the result is true for $n = k + 1$ if it is also true for $n = k$

Exercise 6N;
3bdf, 4 ab(i), 9

Step 4: Since the result is true for $n = 1$, then the result is true for all positive integral values of n by induction.