Mathematical Induction

$e.g.(v)$ Prove $2^n > n^2$ for $n > 4$

Step 1: Prove the result is true for $n = 5$

$LHS = 2^5$ $\qquad$ $RHS = 5^2$

$= 32$ $\qquad$ $= 25$

$\therefore LHS > RHS$

Hence the result is true for $n = 5$

Step 2: Assume the result is true for $n = k$, where k is a positive integer > 4

$i.e.\ 2^k > k^2$

Step 3: Prove the result is true for $n = k + 1$

$i.e.$ Prove: $2^{k+1} > (k+1)^2$

Proof:

$$2^{k+1} = 2 \cdot 2^k$$
$$> 2k^2$$
$$= k^2 + k^2$$
$$= k^2 + k \times k$$
$$> k^2 + 4k \qquad (\because k > 4)$$
$$= k^2 + 2k + 2k$$
$$> k^2 + 2k + 8 \qquad (\because k > 4)$$
$$> k^2 + 2k + 1$$
$$= (k+1)^2$$
$$\therefore 2^{k+1} > (k+1)^2$$

Exercise 6N;
6 abc, 8a, 15

Hence the result is true for $n = k + 1$ if it is also true for $n = k$

Step 4: Since the result is true for $n = 5$, then the result is true for all positive integral values of $n > 4$ by induction .