Complex Numbers

Solving Quadratics

$$x^2+1=0$$

$$x^2=-1$$

$$\text{no real solutions}$$

In order to solve this equation we define a new number

$$i=\sqrt{-1} \quad \text{or} \quad i^2=-1$$

$$i \text{ is an imaginary number}$$

$$x^2=-1$$

$$x=\pm\sqrt{-1}$$

$$x=\pm i$$

$$e.g.\quad x^2+3x+7=0$$

$$x=\frac{-3\pm\sqrt{9-28}}{2}$$

$$=\frac{-3\pm\sqrt{-19}}{2}$$

$$=\frac{-3\pm\sqrt{19}i}{2}$$

$$x=\frac{-3+\sqrt{19}i}{2}\ \text{ or }\ x=\frac{-3-\sqrt{19}i}{2}$$

OR

$$x^2+3x+7=0$$

$$\left(x+\frac{3}{2}\right)^2+\frac{19}{4}=0$$

$$\left(x+\frac{3}{2}\right)^2-\frac{19}{4}i^2=0$$

$$\left(x+\frac{3}{2}+\frac{\sqrt{19}}{2}i\right)\left(x+\frac{3}{2}-\frac{\sqrt{19}}{2}i\right)=0$$

$$x=-\frac{3}{2}+\frac{\sqrt{19}}{2}i\ \text{ or }\ x=-\frac{3}{2}-\frac{\sqrt{19}}{2}i$$

All complex numbers (z) can be written as; $z = x + iy$

Definitions;

(1) All complex numbers contain a **real** and an **imaginary** part

$$\operatorname{Re}(z) = x$$
$$\operatorname{Im}(z) = y$$

e.g. $z = 3 + 5i$

$\operatorname{Re}(z) = 3$ $\operatorname{Im}(z) = 5$

(2) If Re(z) = 0, then z is a **pure imaginary number**

e.g. $\sqrt{3}i, -6i$

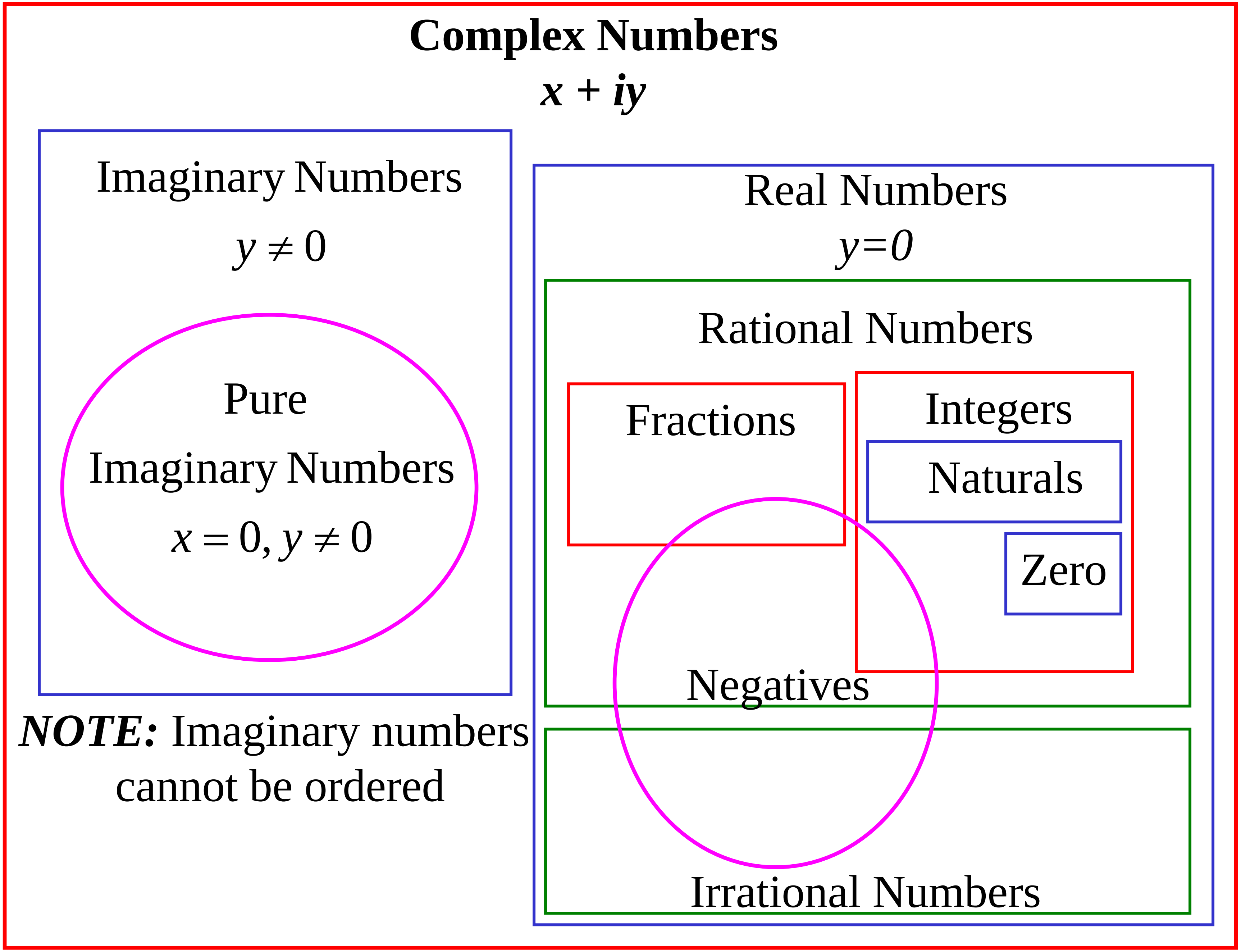
(3) If Im(z) = 0, then z is a **real number**

e.g. $\frac{3}{4}, \pi, e, -4$

(4) Every complex number $z = x + iy$, has a **complex conjugate**

$$\bar{z} = x - iy$$

e.g. $z = -2 - \sqrt{7}i$ $\bar{z} = -2 + \sqrt{7}i$


Complex Numbers
x + iy
Imaginary Numbers
y ≠ 0
Pure
Imaginary Numbers
x = 0, y ≠ 0
NOTE: Imaginary numbers
cannot be ordered
Real Numbers
y=0
Rational Numbers
Fractions
Integers
Naturals
Zero
Negatives
Irrational Numbers

Basic Operations

As *i* a surd, the operations with complex numbers are the same as surds

Addition

$$(4-3i)+(-8+2i)$$

$$=-4-i$$

Subtraction

$$(4-3i)-(-8+2i)$$

$$=12-5i$$

Multiplication

$$(4-3i)(-8+2i)$$

$$=-32+8i+24i-6i^2$$

$$=-32+32i+6$$

$$=-26+32i$$

Division (Realising The Denominator)

$$\frac{(4-3i)}{(-8+2i)}\times\frac{(-8-2i)}{(-8-2i)}$$

$$=\frac{-32-8i+24i-6}{64+4}$$

$$=\frac{-38+16i}{68}$$

$$=\frac{-19}{34}+\frac{8}{34}i$$

Exercise 4A; 1 to 16 evens