Complex Equations

If two complex numbers are equal, then their real parts are equal and their imaginary parts are equal

$$i.e. \text{ If } a_1 + b_1 i = a_2 + b_2 i$$

then

$$a_1 = a_2$$

$$b_1 = b_2$$

$$e.g.(i) \quad x + iy = (2+3i)(4-2i)$$

$$= 8 - 4i + 12i + 6$$

$$= 14 + 8i$$

$$\therefore x = 14 \ , \ y = 8$$

(ii) $z+2iw=4+3i$

$2z+iw=3+4i$

$$\Rightarrow \begin{array}{r} z+2iw=4+3i \\ 4z+2iw=6+8i \\ \hline 3z \qquad\quad =2+5i \end{array}$$

$$z=\frac{2}{3}+\frac{5}{3}i$$

$$\therefore \frac{2}{3}+\frac{5}{3}i+2iw=4+3i$$

$$2iw=\frac{10}{3}+\frac{4}{3}i$$

$$w=\frac{10}{6i}+\frac{4}{6}$$

$$=\frac{2}{3}-\frac{5}{3}i$$

$$w=\frac{2}{3}-\frac{5}{3}i \quad , \quad z=\frac{2}{3}+\frac{5}{3}i$$

(iii) Find the quadratic equation with roots $4+i$ and $4-i$

$$\alpha+\beta=8 \qquad \alpha\beta=17$$

$$\therefore \text{equation is } x^2-8x+17=0$$

(iv) $a+ib=\sqrt{5-12i}$

$$(a+ib)^2=5-12i$$

$$a^2+2abi-b^2=5-12i$$

$$\therefore a^2-b^2=5 \qquad 2ab=-12$$

$$b=-\frac{6}{a}$$

$$a^2-\left(-\frac{6}{a}\right)^2=5$$

$$a^2-\frac{36}{a^2}=5$$

$$a^4-5a^2-36=0$$

$$(a^2-9)(a^2+4)=0$$

$$a^2=9 \quad \text{or} \quad a^2=-4$$

$$a=\pm 3 \qquad \text{no real solutions}$$

$$\therefore b=\mp 2$$

$$\therefore a=3, b=-2 \quad \text{or} \quad a=-3, b=2$$

$$\text{If } \sqrt{x+iy} = a+ib$$
$$\text{then } a^2 - b^2 = x$$
$$2ab = y$$

$e.g.$ Find $\sqrt{-12+16i}$

$$a^2 - b^2 = -12$$

$$2ab = 16$$

$$b = \frac{8}{a}$$

$$a^2 - \frac{64}{a^2} = -12$$

$$a^4 + 12a^2 - 64 = 0$$

$$(a^2 - 4)(a^2 + 16) = 0$$

$$a^2 = 4 \quad \text{or} \quad a^2 = -16$$
$$a = \pm 2 \qquad \text{no real solutions}$$

$$\therefore b = \pm 4$$

$$\sqrt{-12+16i} = \pm(2+4i)$$

Exercise 4A; 17 to 21, 22a, 23bc, 24, 25bd, 26ac, 27acd, 28ac, 29abd

Exercise 4G; 1 aceg, 3 to 7