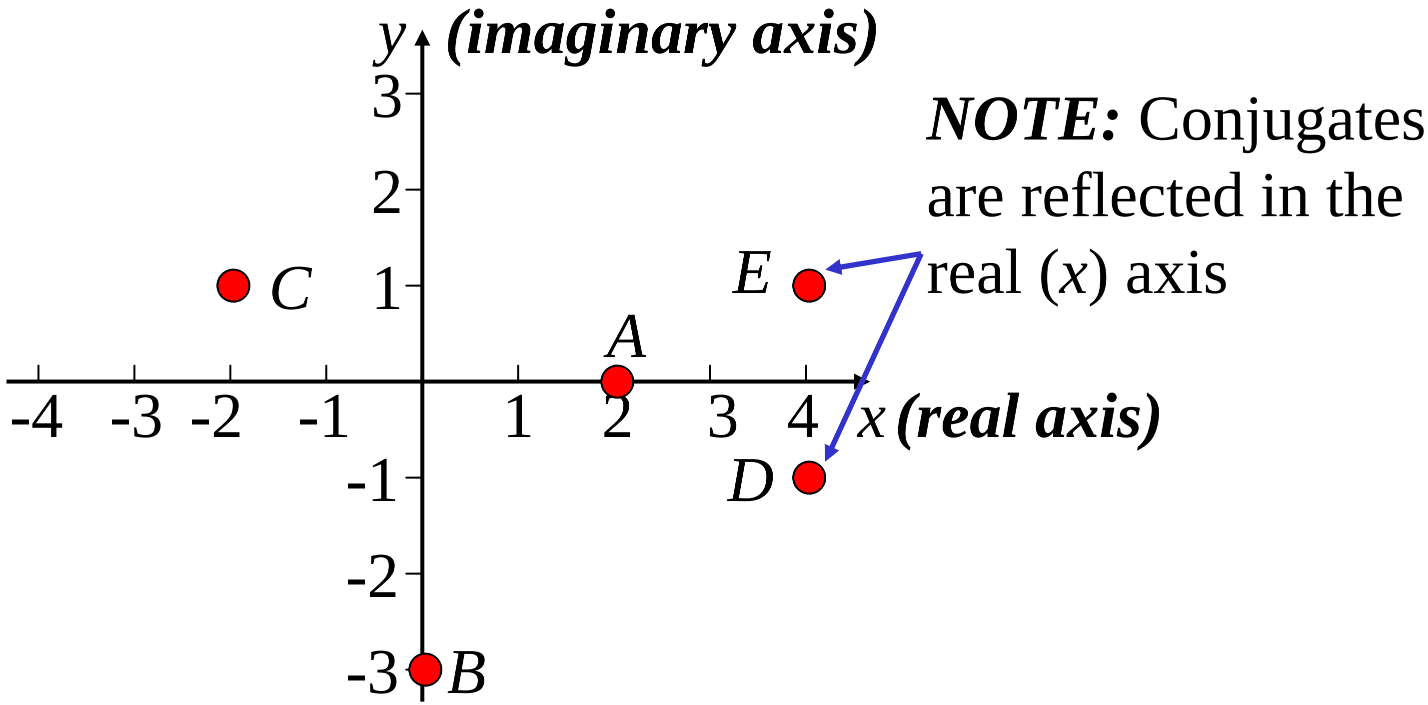
The Argand Diagram

Complex numbers can be represented geometrically on an **Argand Diagram.**



$A = 2$

$B = -3i$

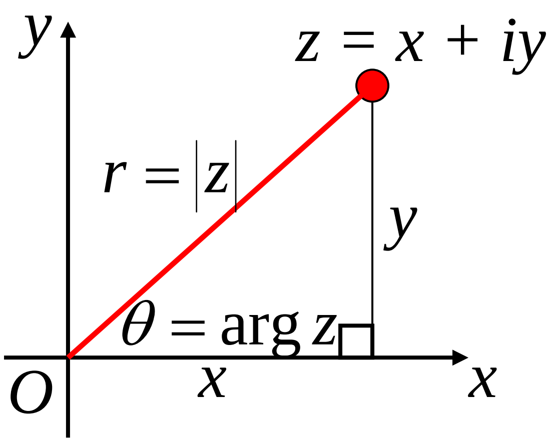
$C = -2 + i$

$D = 4 - i$

$E = 4 + i$

Every complex number can be represented by a unique point on the Argand Diagram.

Mod-Arg Form



Modulus

The modulus of a complex number is the length of the vector *OZ*

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$|z| = \sqrt{x^2 + y^2}$$

Argument

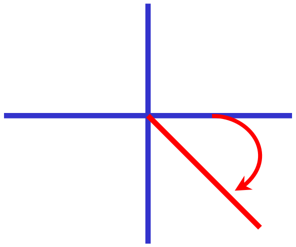
The argument of a complex number is the angle the vector *OZ* makes with the positive real (x) axis

$$\arg z = \tan^{-1}\left(\frac{y}{x}\right) \qquad -\pi < \arg z \leq \pi$$

$e.g.$ Find the modulus and argument of $4-4i$

$$|4-4i| = \sqrt{4^2+(-4)^2}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

$$\arg(4-4i) = \tan^{-1}\left(\frac{-4}{4}\right)$$
$$= \tan^{-1}(-1)$$
$$= -\frac{\pi}{4}$$



Every complex number can be written in terms of its modulus and argument

$$z = x+iy$$
$$= r\cos\theta + ir\sin\theta$$
$$= r(\cos\theta + i\sin\theta)$$

The **mod-arg** form of *z* is;

$$z = r(\cos\theta + i\sin\theta)$$
$$z = r cis\theta$$

where; $r = |z|$

$$\theta = \arg z$$

$$e.g.\ (i)\ 4-4i = 4\sqrt{2}cis\left(-\frac{\pi}{4}\right)$$

$$(ii)\ \sqrt{3}+i$$

$$|z| = \sqrt{\left(\sqrt{3}\right)^2+1^2}$$
$$= \sqrt{4}$$
$$= 2$$

$$\arg z = \tan^{-1}\frac{1}{\sqrt{3}}$$
$$= \frac{\pi}{6}$$

$$\therefore \sqrt{3}+i = 2cis\frac{\pi}{6}$$

Exercise 4B; evens