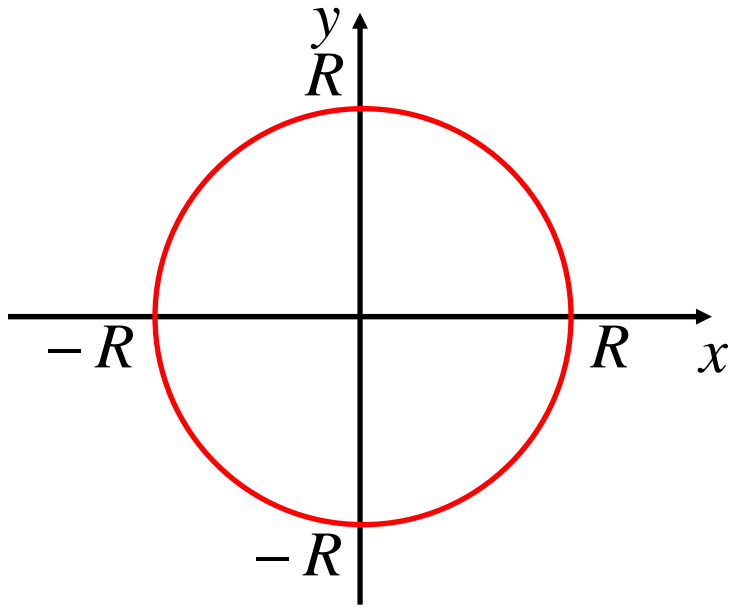


Locus and Complex Numbers

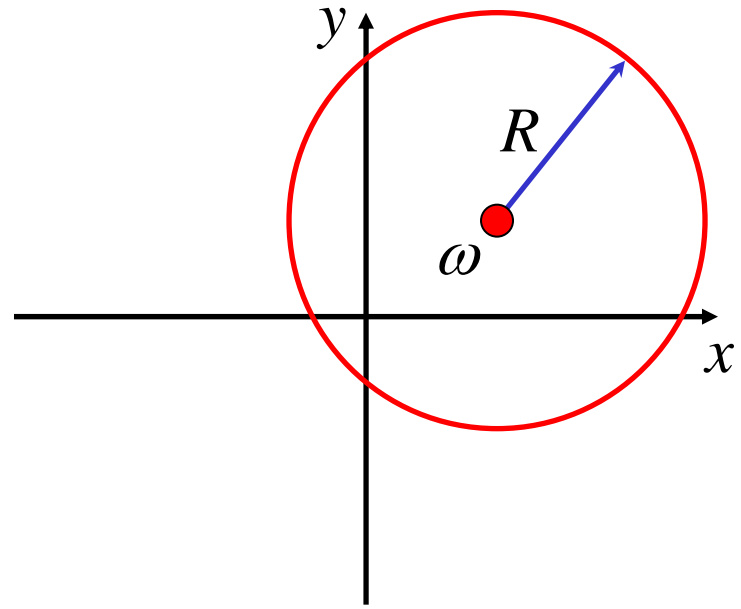
Circles



$$|z| = R$$

or

$$z\bar{z} = R^2$$



$$|z - \omega| = R$$

or

$$(z - \omega)(\bar{z} - \bar{\omega}) = R^2$$

e.g. (i) Express these circles in terms of z

a) $x^2 + y^2 = 16$

$$\underline{|z| = 4}$$
$$\underline{(z\bar{z} = 16)}$$

b) $x^2 + y^2 + 6x - 4y - 12 = 0$

$$x^2 + 6x + y^2 - 4y = 12$$

$$(x + 3)^2 + (y - 2)^2 = 25$$

$$\underline{|z + 3 - 2i| = 5}$$

$$[(z + 3 - 2i)(\bar{z} + 3 + 2i) = 25]$$

(ii) Find the centre and radius of;

a) $|z - 5 - i| = 2$

$$\underline{\text{centre : } (5, 1)}$$

$$\underline{\text{radius : } 2 \text{ units}}$$

b) $(z + 4 + i)(\bar{z} + 4 - i) = 49$

$$\underline{\text{centre : } (-4, -1)}$$

$$\underline{\text{radius : } 7 \text{ units}}$$

$$\text{c) } |3z| = |z + 2 - i|$$

$$3|z| = |z + 2 - i|$$

$$9x^2 + 9y^2 = (x + 2)^2 + (y - 1)^2$$

$$9x^2 + 9y^2 = x^2 + 4x + 4 + y^2 - 2y + 1$$

$$8x^2 - 4x + 8y^2 + 2y = 5$$

$$x^2 - \frac{1}{2}x + y^2 + \frac{1}{4}y = \frac{5}{8}$$

$$\left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{8}\right)^2 = \frac{45}{64}$$

$$\text{centre: } \left(\frac{1}{4}, -\frac{1}{8}\right)$$

$$\text{radius: } \frac{3\sqrt{5}}{8} \text{ units}$$

$$\text{d) } z\bar{z} + 2(z + \bar{z}) = 0$$

$$x^2 + y^2 + 4x = 0$$

$$(x + 2)^2 + y^2 = 4$$

$$\text{centre: } (-2, 0)$$

$$\text{radius: } 2 \text{ units}$$

**Exercise 4M; 1ac, 2bd, 3, 4,
5bd, 6**