Logarithms

Logarithms are the inverse of exponentials.

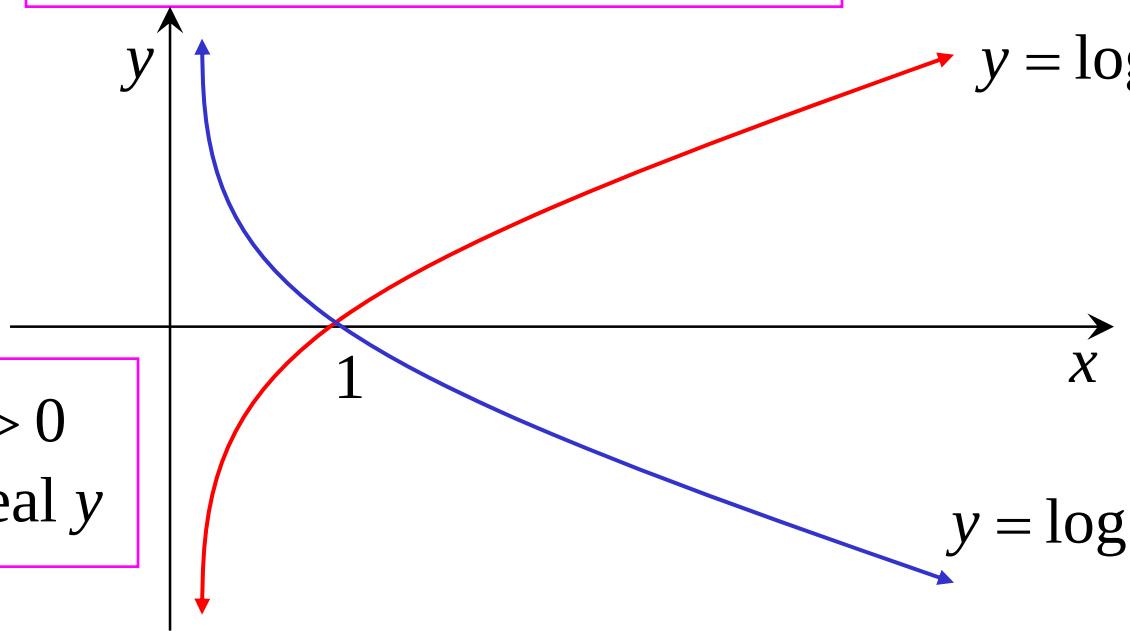
If $y = a^x$ then $x = \log_a y$

If $y = e^x$ then $x = \log_e y$

$x = \ln y$

$x = \log y$

log base e is known as the natural logarithm.



$y = \log_a x \qquad (a > 1)$

$y = \log_a x \qquad (0 < a < 1)$

domain : $x > 0$

range : all real y

Log Laws

(1) $\log_a m + \log_a n = \log_a mn$

(2) $\log_a m - \log_a n = \log_a \left(\frac{m}{n} \right)$

(3) $\log_a m^n = n \log_a m$

(4) $\log_a 1 = 0$

(5) $\log_a a = 1$

(6) $a^{\log_a x} = x$

(7) $\log_a x = \frac{\log_b x}{\log_b a}$

e.g. (i) $x = \log_5 125$

$5^x = 125$

$x = 3$

(ii) $\log_x 343 = 3$

$x^3 = 343$

$x = 7$

(iii) Evaluate;

a) $\log_4 16$

$= \log_4 4^2$

$= 2\log_4 4$

$= 2$

b) $6^{2\log_6 3}$

$= 6^{\log_6 3^2}$

$= 3^2$

$= 9$

c) $\log_2 16 + \log_2 8$

$= \log_2 128$

$= \log_2 2^7$

$= 7$

d) $\log_{10} 125 + \log_{10} 32 - \log_{10} 4$

$= \log_{10}\left\{\frac{125 \times 32}{4}\right\}$

$= \log_{10} 1000$

$= 3$

e) $\frac{\log_7 8}{\log_7 2}$

$= \log_2 8$

$= 3$

f) $\log_2 \sqrt{\frac{1}{8}}$

$= \frac{1}{2}\log_2 \frac{1}{8}$

$= \frac{1}{2}(-3)$

$= \frac{-3}{2}$

$(iv)\quad 3^{2x+1} = \dfrac{1}{27}$

$$3^{2x+1} = 3^{-3}$$
$$2x+1 = -3$$
$$2x = -4$$
$$x = -2$$

$(v)\quad 2^x = 9$

$$\log 2^x = \log 9$$
$$x \log 2 = \log 9$$
$$x = \frac{\log 9}{\log 2}$$
$$x = 3.17 \quad \text{(to 2 dp)}$$

Exercise 12A; 2, 3aceg, 4bdfh, 5ab, 6ab, 7ac, 8bdh, 9ac, 14, 18*

Exercise 6B; 8