

Differentiating Logarithms

$$y = \log f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\log a)f(x)}$$

e.g. (i) $y = \log(3x+5)$

$$\frac{dy}{dx} = \frac{3}{3x+5}$$

(ii) $y = \log x^3$

$$\begin{aligned} \frac{dy}{dx} &= \frac{3x^2}{x^3} \\ &= \frac{3}{x} \end{aligned}$$

OR

$$y = \log x^3$$

$$y = 3 \log x$$

$$\frac{dy}{dx} = \frac{3}{x}$$

$$(iii) \quad y = \log(\log x)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x} \\ &= \frac{1}{x \log x} \end{aligned}$$

$$(iv) \quad y = \log(x+3)(x+2)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+3)(1) + (x+2)(1)}{(x+3)(x+2)} \\ &= \frac{2x+5}{(x+3)(x+2)} \end{aligned}$$

OR

$$y = \log(x+3) + \log(x+2)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{x+3} + \frac{1}{x+2} \\ &= \frac{(x+2) + (x+3)}{(x+3)(x+2)} \\ &= \frac{2x+5}{(x+3)(x+2)} \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad y &= \log \left\{ \frac{x+5}{x+2} \right\} \\
 \frac{dy}{dx} &= \frac{(x+2)(1) - (x+5)(1)}{(x+2)^2} \\
 &= \frac{-3}{(x+2)^2} \times \frac{(x+2)}{(x+5)} \\
 &= \frac{-3}{(x+2)(x+5)}
 \end{aligned}$$

OR

$$\begin{aligned}
 y &= \log(x+5) - \log(x+2) \\
 \frac{dy}{dx} &= \frac{1}{x+5} - \frac{1}{x+2} \\
 &= \frac{(x+2) - (x+5)}{(x+5)(x+2)} \\
 &= \frac{-3}{(x+2)(x+5)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad y &= \log_{10} 6x \\
 \frac{dy}{dx} &= \frac{6}{(\log 10)6x} \\
 &= \frac{1}{x \log 10}
 \end{aligned}$$

Exercise 12B; 1acf, 2chk, 5acehi, 6b, 7ad, 8acef, 9bd, 10ac, 11, 13a, 14bdfhjl, 15b, 18bdf, 19b, 20af*, 21a*

Exercise 12C; 1bdf, 2, 3, 6, 7a, 8, 11, 13, 14, 18*