Integrating Derivative on Function

$$\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + c$$

e.g. (*i*) $\displaystyle\int \frac{1}{7-3x}\,dx$

$$= -\frac{1}{3}\int \frac{-3}{7-3x}\,dx$$

$$= -\frac{1}{3}\log(7-3x) + c$$

(*ii*) $\displaystyle\int \frac{dx}{8x+5}$

$$= \frac{1}{8}\int \frac{8dx}{8x+5}$$

$$= \frac{1}{8}\log(8x+5) + c$$

(*iii*) $\displaystyle\int \frac{x^5}{x^6-2}\,dx$

$$= \frac{1}{6}\int \frac{6x^5}{x^6-2}\,dx$$

$$= \frac{1}{6}\log\left(x^6-2\right) + c$$

$$(iv)\ \int \frac{1}{5x}\,dx \qquad \textbf{\textit{OR}} \qquad \frac{1}{5}\int \frac{1}{x}\,dx$$

$$= \frac{1}{5}\int \frac{5}{5x}\,dx \qquad\qquad = \frac{1}{5}\log x + c$$

$$= \frac{1}{5}\log 5x + c$$

$$(v)\ \int \frac{4x+1}{2x+1}\,dx$$

order numerator $\geq$ order denominator

$\Rightarrow$ polynomial division

$$= \int \left[2 - \frac{1}{2x+1}\right] dx$$

$$= 2x - \frac{1}{2}\log(2x+1) + c$$

$$\begin{array}{r} 2 \\ 2x+1\,\overline{)\,4x+1} \\ 4x+2 \\ \hline -1 \end{array}$$

$(vi)\quad \int_1^2 \frac{2x}{x^2+1}dx$

$= \left[\log\left(x^2+1\right)\right]_1^2$

$= \log 5 - \log 2$

$= \log\left(\frac{5}{2}\right)$

(vii) Differentiate $x^3 \log x$ and hence integrate $x^2 \log x$

$$\frac{d}{dx}\left\{x^3 \log x\right\} = \left(x^3\right)\left(\frac{1}{x}\right) + (\log x)\left(3x^2\right)$$

$$= x^2 + 3x^2 \log x$$

$$\int \left(x^2 + 3x^2 \log x\right)dx = x^3 \log x + c$$

$$3\int x^2 \log x dx = x^3 \log x - \int x^2 dx + c$$

$$\int x^2 \log x dx = \frac{1}{3}x^3 \log x - \frac{1}{9}x^3 + c$$

Exercise 12D; 1 to 12 ace in all, 14a*

Exercise 12E; 1 to 6 all, 7 to 21 odds, 22abc*, 23*